Credit Crunches, Individual Heterogeneity and the Labor Wedge

December 16, 2016

Abstract

Standard neoclassical theory suggests that the marginal product of labor (MPL) should be equal to the marginal rate of substitution between consumption and leisure (MRS). Yet this is not the case in the data. Understanding the measured discrepancy between the MPL and the MRS, commonly referred as *the labor wedge*, is important to comprehend the limitations of economic models and thereafter improve them. The labor wedge has increased significantly during the Great Recession, but the mechanism of its variation in the credit crunch have not been well understood. This paper fills in this gap by studying the labor wedge in a quantitative general equilibrium model with collateral borrowing constraints. I find that a credit crunch can affect the labor wedge through a mechanism different from that of an exogenous TFP shock when there is endogenous entry and exit of production. With entry and exit, the tightening of the collateral constraints can cause the gap between the real wage and the MRS to increase, but the exogenous TFP shock does not have this mechanism. As a result, the labor wedge has higher increases in the credit crunch. When entry and exit is shut down, the labor wedge would have much smaller increases in the credit crunch.

JEL Classification: E21, E23, E24, E32
Keywords: Financial frictions, labor wedge, entry and exit, business cycle

1 Introduction

When analyzing household time allocation decision, standard neoclassical theory predicts that the optimal time allocation is achieved when the marginal product of labor (MPL) equals the marginal rate of substitution between consumption and leisure (MRS). In the data, the equality between the MPL and the MRS typically does not hold. Thus the discrepancy measured in the data between the MPL and the MRS, commonly referred to as *the labor wedge*, has attracted growing attention. Understanding the mechanism causing the labor wedge and its variation over the business cycle can help economists to comprehend the limitation of economic models and thereafter improve the models to better explain economic activity.

Examining the fluctuations of the labor wedge over the U.S. business cycle in Figure 1 yields two observations. First, the labor wedge is countercyclical. It declines in economic booms and increases during recessions. Second, although the decline in output is deeper in the 1981-1982 recession, the increase of labor wedge is larger in the 2007-2009 financial crisis than that in other U.S. recessions. The countercyclicality of the labor wedge has been widely established in the literature.[1] Nevertheless, the higher increase in the labor wedge during the Great Recession has not been fully studied. Particularly, it is not clear whether the credit crunch affects the labor wedge in a mechanism different from that of other sources of business cycle fluctuations and hence the variation of the labor wedge follows to be different. The objective of this paper is to fill in this gap and explore the impact of the credit crunch on the labor wedge.

Departing from the traditional approach of modeling labor market failures,[2] this paper builds a novel model which allows endogenous entry and exit of production and collateral borrowing constraints to jointly affect the movements of the labor wedge over the business cycle. The labor wedge may arise endogenously in the absence of labor market distortions due to worker and producer heterogeneity as in Chang and Kim (2007) and Buera and Moll (2015). Moreover, the fluctuations of the labor wedge in the paper is mostly attributable to the gap between the real wage and the MRS. This pattern is notified in Karabarbounis (2014)[3] and Bils, Klenow and Malin (2015) and shown in the bottom panel of Figure 1.

Specifically, the model economy has a continuum of individuals with heterogeneous entrepreneurial abilities and assets. In each period, individuals face discrete occupational

[1]Karabarbounis (2014), Bils, Klenow and Malin (2015), Chahrour et al (2016), to name a few.

[2]For example, Shimer (2008), Ohanian (2010), Epstein and Ramnath (2012), Chahrour et al (2016), Pescatori and Tasci (2011).

[3]Karabarbounis (2014) says that "Remarkably, in the United States, the gap between the measured MPL and the real wage explains on its own at most 2 percent of variation of the labor wedge over the business cycle. By contrast, the gap between the real wage and the MRS explains on its own roughly 80 percent of the cyclical variation of the labor wedge."

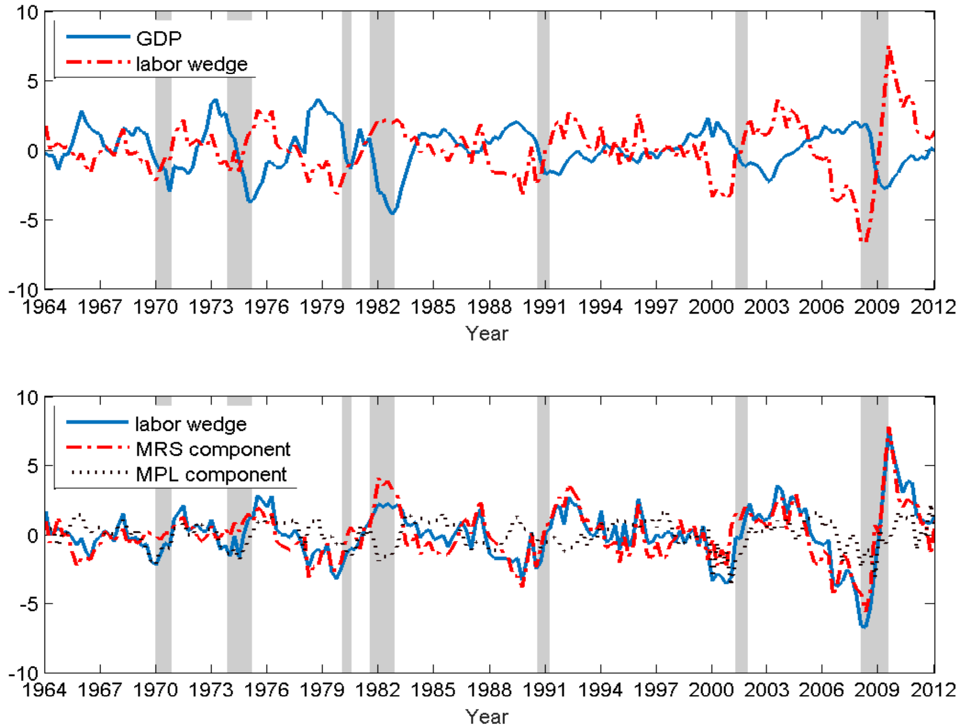

Figure 1: The cyclical variations of real GDP and the labor wedge.

Note.– The bottom panel shows the decomposition of the cyclical component of the labor wedge. The real GDP and the labor wedge are HP-filtered series with smoothing parameter 1600. Shaded areas represent U.S. recessions.

choices as workers or entrepreneurs. Workers supply labor flexibly for wages while entrepreneurs work a fixed amount of time. Entrepreneurs can make profits from production with capital and labor as inputs. Each entrepreneur's capital input is subject to a collateral borrowing constraint, which limits the maximum capital input based on the entrepreneur's asset. A credit crunch in the model is an unexpected reduction of the maximum amount of capital that entrepreneurs can borrow from the financial markets.

Under the framework, the labor wedge can arise due to worker and producer heterogeneity via two channels or both. The first channel is when the underlying individual heterogeneity is such that the real wage deviates from the *aggregate* marginal rate of substitution between consumption and leisure (MRS). This could happen when the real wage does not equal to the marginal rate of substitution of a portion of individuals in the economy. Consequently, a gap between the real wage and the MRS would exist after aggregation of all individuals. In the model, this portion of individuals are entrepreneurs and voluntarily unemployed workers. The second channel is when individual heterogeneity is such that the *aggregate* marginal product of labor (MPL) deviates from the real wage. This could happen when a portion of entrepreneurs in the economy do not hire labor from the labor market. In the benchmark economy, however, the measure of this portion of entrepreneurs are very small so that the MPL approximately equals the real wage. As a result, the labor wedge arises in the benchmark economy via the first channel— there is a gap between the real wage and the MRS. Specifically, the deviation of the real wage from the MRS is consisted of two components: an entrepreneurial deviation which represents the deviation of the real wage from the entrepreneurs' aggregate marginal rate of substitution (MRS_e) and a workers' deviation which represents the gap between the real wage and the aggregate marginal rate of substitution of voluntarily unemployed people (MRS_u).

I find that when the economy has endogenous entry and exit of production, a credit crunch can affect the labor wedge through a mechanism different from that of an exogenous TFP shock — a more standard source of business cycle fluctuations. Consequently, the labor wedge has higher increases in the credit crunch. The core mechanism is that the tighter collateral constraints in the credit crunch affect entrepreneurs' consumption and opportunity cost of leisure in different patterns, but the exogenous TFP shock does not have this channel. On the one hand, the tighter collateral constraints reduce constrained entrepreneurs' profits and consumption, but unconstrained entrepreneurs expand production for higher profits and consumption as the factor prices are lower in the shock. The exits of low consumption constrained entrepreneurs and the presence of higher consumption unconstrained entrepreneurs allow entrepreneurs' aggregate consumption to recover since year 2 of the shock. On the other hand, the tighter collateral constraints limit capital input and keep the MPL low. With the declining entrepreneurial population and real wage, entrepreneurs' opportunity cost of leisure keeps falling. With the CRRA utility function, the deviation of the real wage from the MRS is proportional to the ratio of consumption and opportunity cost of leisure. Thus when entrepreneurs' consumption and opportunity cost of leisure move in opposite directions, the deviation of the real wage from the MRS_e increases. It follows that the gap between the real wage and the MRS increases after aggregation. As a result, the labor wedge has larger increases in the credit crunch. In the TFP shock, however, entrepreneurs' consumption and opportunity cost of leisure do not move in opposite directions persistently. Therefore, the deviation of the real wage from the MRS is smaller in the TFP shock.

The paper also contributes to study the impact of endogenous entry and exit of production on the labor wedge without labor market distortions. Recently, entry and exit has been found to amplify aggregate fluctuations (Clementi and Palazzo 2016) and economic growth

(Asturias et al 2016). While reproducing the amplification effect on aggregate fluctuations, this paper shows that entry and exit also amplifies the variation of the labor wedge in a credit crunch. When entrepreneurial entry and exit is shut down, the labor wedge would have much smaller increases in a credit crunch. Without entry and exit, entrepreneurs' consumption only recovers slowly as constrained entrepreneurs cannot exit. Also, their opportunity cost of leisure stop declining since year 2 of the shock due to the absence of entry and exit. As a result, the deviation of the real wage from the MRS and hence the labor wedge remains small throughout the credit crunch. To match the variation of the labor wedge in the Great Recession, endogenous entry and exit of production may be an essential element to include in economic models.

1.1 Related Literature

The paper is first related to the literature that use aggregate data and wedges in the representative agent framework to analyze the business cycle. The seminal work by Chari, Kehoe and McGrattan (2007) find that investment wedge did not fluctuate much over the business cycle in the postwar aggregate data and hence models in which financial frictions as in Carlstrom and Fuerst (1997) and Bernanke, Gertler and Gilchrist (1989) that show up as investment wedges may not be promising, but models with financial frictions that show up as the efficiency or labor wedges may well be promising. However, Christiano and Davis (2006), Justiniano, Primiceri and Tambalotti (2010, 2011) point out that investment wedge is an important source of business cycle fluctuations.

A large number of research relates the labor wedge to labor market distortions. Leading examples are Shimer (2008), Ohanian (2010), Epstein and Ramnath (2012), Chahrour et al (2016), Pescatori and Tasci (2011) and Cheremukhin and Restrepo-Echavarria (2008). Nevertheless, Bils, Klenow and Marlin (2015) and Duras (2015) criticizes the shortcoming of the labor search model and shows that good market frictions also plays important role for the labor wedge. Departing from this literature, I study the labor wedge without labor or good markets frictions.

Several papers have found that the labor wedge can arise when individual heterogeneity in the economy is considered. Leading examples are Chang and Kim (2007), Cociuba and Ueberfeldt (2015), and Buera and Moll (2015). Chang and Kim (2007) find that the labor wedge can arise endogenously without labor market distortions. A key difference between my paper and theirs is that I focus on the variation of the labor wedge in a credit crunch while they use the existence of labor wedge to explain the low correlation between hours and productivity using only TFP shocks. Buera and Moll (2015) find that a credit crunch shows up as a different wedge in a different model variant, thus wedges may not be used to identify the sources of aggregate fluctuations. My study differs from theirs in two dimensions. First, they focus on the aggregate implications of a credit crunch while I focus on explaining the mechanism of the labor wedge variation in a credit crunch. Second, they allow entrepreneurs to be inactive without any cost when productivity is low and assume entrepreneurs are absent from uncertainty in their future individual productivity when they make consumption and production decisions. Under these assumptions, a credit crunch is isomorphic to a negative TFP shock in their model. My analysis considers a more common form of entry and exit of production and assumes that entrepreneurs are uncertain about their future individual productivity. Thus the effects of the credit crunch and the negative TFP shock on the labor wedge are essentially different.

A number of papers have found that limited enforceability in firm financing conditions can generate the labor wedge. Leading examples are Jermann and Quadrini (2012) and Arellano, Bai and Kehoe (2012). My work is complementary to this literature but novel in two aspects. First, I focus on explaining the fluctuations of the labor wedge in a credit crunch while they study the real aggregate effects of a credit crunch (Jermann and Quadrini (2012)) and uncertainty shocks (Arellano, Bai and Kehoe (2012)). Second, the labor wedge arises endogenously in my model due to individual heterogeneity while they find that the labor wedge could arise because the limited enforeability in debt contracts make firms hire less labor.

The paper is also related to the literature that emphasize the role of financial frictions in a heterogeneous agent model including Buera, Kaboski and Shin (2011), Buera and Shin (2013), Buera, Fattel-Jaef and Shin (2015) and Bassetto, Cagetti and De Nardi (2015). Particularly, Buera, Fattel-Jaef and Shin (2015) explains the severe decline of hours over the Great Recession. Although my model shares common elements with theirs, I depart from their objective to study the effects of a credit crunch and endogenous entry and exit of production on the fluctuations of the labor wedge over the business cycle.

The rest of the paper is organized as follows. Section 2 describes the baseline model. Section 3 presents parametrization. Section 4 summarizes the properties of the economy in the steady state. Section 5 discusses the real effects of a credit crunch and compares it to an exogenous TFP shock. Then it presents the findings in a simpler model without entry and exit. Section 6 concludes.

2 The Model

2.1 Heterogeneity and Demographics

Time is discrete. The economy is populated by a continuum of individuals indexed by $i \in [0,1]$.[4] Each individual is endowed with one unit of time to divide between labor and leisure. Individuals live indefinitely and can save using risk-free assets a.

Individuals are endowed with idiosyncratic entrepreneurial ability θ, which measures their efficiency of managing production using capital and labor. Entrepreneurial productivity evolves according to an exogenous Markov process P_θ. The entrepreneurial ability is inalienable and there is no market for this ability.

In each period, individuals face discrete occupational choices. At the beginning of a period, each individual draws a random fixed cost ξ, denoted in units of consumption good, from a uniform distribution $G(\xi)$ where $\xi \in [0, \xi_U]$. After the realization of the fixed cost ξ, each individual chooses whether to pay the realized fixed cost to change occupation (entrepreneur or worker) or maintain his previous job and pay nothing. That is, the idiosyncratic random fixed cost is only paid when an individual changes occupation.

The discrete occupational choices allow entrepreneurs to endogenously enter and exit production sector. Since the fixed cost is only paid when an individual switches occupation, keeping track of the occupational history for one period is sufficient for the analysis. Let χ_{-1} denote the occupational history of an individual. Then,

$$\chi_{-1} = \begin{cases} 1, & \text{if last period he is an entrepreneur} \\ 0, & \text{if last period he is a worker} \end{cases} \tag{1}$$

Let $\mu(a, \theta, \chi_{-1})$ denote the cumulative distribution of individual wealth a, entrepreneurial ability θ and occupational history χ_{-1} across individuals in the economy.

2.2 Preference

All individuals derive utility from consumption and leisure and discount their future utility using discount factor β. Individual i's utility function for consumption and leisure is given by

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t, 1 - n_t), \qquad u(c, 1-n) = \frac{\left(c^\tau (1-n)^{(1-\tau)}\right)^{1-\sigma} - 1}{1-\sigma} \tag{2}$$

[4] In the paper I use capital letters to denote aggregates and lowercase letters to denote variables at the individual level. The individual index i is left implicit in the context whenever there is no confusion.

Entrepreneurs and workers share the same form of utility function as in equation (2).

2.3 Technology

An entrepreneur with talent θ works $\bar{n}$ unit of time for himself and produces with capital and labor according to the following production technology

$$f(z, \theta, k, l) = z\theta(k^{\alpha}(\bar{n} + l)^{1-\alpha})^{\nu} \tag{3}$$

where z is aggregate productivity, k is capital input and l is labor hired from the labor market. Parameter $\alpha \in (0, 1)$ captures the fraction of entrepreneurial income that goes to capital. Parameter $\nu \in (0, 1)$ captures decreasing returns to scale (DRS) technology because an entrepreneur's management skill might gradually become insufficient as production scale increases.

The labor market is perfectly competitive and frictionless. All individuals in the economy face the same real wage w and real interest rate r with the belief that their occupational choices, asset decisions and labor supply do not affect prices.

2.4 Financial Markets

Financial intermediaries are perfectly competitive. At the beginning of each period after the entrepreneurial ability and fixed cost are realized and the occupational choices are made, workers and entrepreneurs deposit assets a in the financial markets. The return on the deposited assets is the real interest rate r. Thereafter, entrepreneurs borrow capital from financial intermediaries using their deposited assets a as collateral. The amount of capital that each entrepreneur can borrow is limited by the linear collateral constraint, $k \leq \lambda a$, where the parameter λ is the same for every entrepreneur and it measures the degree of financial market incompleteness. Specifically, $\lambda = 1$ is financial autarky and $\lambda = \infty$ is perfect financial market. Since capital is assumed to depreciate at rate δ, the effective capital rental rate is $r + \delta$. Financial markets allow intratemporal borrowing of capital for production but forbid intertemporal borrowing to smooth consumption, i.e. $a \geq 0$ applies to all individuals. Thus workers are pure lenders while entrepreneurs can borrow capital for production but can not borrow for smoothing consumption.

Inefficiency arises because the linear borrowing constraints may prevent constrained entrepreneurs with high-ability to borrow up to their optimal capital inputs. In addition, high fixed cost may prevent high ability individuals from entering production.

The timing of the economy is as follows. First, aggregate productivity and individual productivity are realized and known to all households. Second, individuals draw random fixed costs and then the occupational choices are made. Third, individuals deposit assets in the financial market. Entrepreneurs rent capital, hire labor and carry out productions. Finally, workers get paid, entrepreneurs take the profits, individuals trade and consume final goods.

2.5 The individual's problem

Let S denote the aggregate state in the economy, then $S = (z, \mu)$. Let $v^1(a, \chi_{-1}, \theta, \xi, S)$ represent the expected discounted value of an individual with assets a, entrepreneurial ability θ, occupational history χ_{-1} and fixed cost ξ when the aggregate state of the economy is S.

Define $v^0(a, \chi_{-1}, \theta, S)$ as the beginning of period expected value of an individual prior to the realization of its fixed cost, but after the determination of $(a, \chi_{-1}, \theta, S)$. Thus

$$v^0(a, \chi_{-1}, \theta, S) \equiv \int_0^{\xi_U} v^1(a, \chi_{-1}, \theta, \xi, S) G(d\xi)$$

At the beginning of each period, an individual chooses whether to change occupation after the fixed cost ξ is realized. The individual chooses to be an entrepreneur only if running an individual-specific technology gives him higher discounted lifetime value than being a worker. Regardless of his occupational choice, his value function can be expressed as

$$v^1(a, \chi_{-1}, \theta, \xi, S) = \max \{v^e(a, \chi_{-1}, \theta, \xi, S), v^w(a, \chi_{-1}, \theta, \xi, S)\}$$

The entrepreneur's problem

Taking the real wage w and interest rates r as given, every period an entrepreneur with assets a, occupational history χ_{-1}, ability θ and fixed cost ξ solves the following problem:

$$v^e(a, \chi_{-1}, \theta, \xi, S) = \max_{\{c, a', k, l\}} \left\{ u(c, 1 - \bar{n}) + \beta E[v^0(a', 1, \theta', S')] \right\} \tag{4}$$

subject to

$$c + a' \leq f(\theta, z, k, l) - wl - (\delta + r)k + (1 + r)a - \xi(1 - \chi_{-1}) \tag{5}$$

$$k \leq \lambda a \tag{6}$$

$$c > 0, \quad a', k, l \geq 0 \tag{7}$$

Equation (5) is the entrepreneur's budget constraint. Fixed cost ξ is paid when $\chi_{-1} = 0$, i.e. he is a worker last period. On the right hand side, the entrepreneur's revenue is consisted of production profit and asset income. On the left hand side, the entrepreneur allocates his revenue for consumption and saving. Equation (6) is the collateral borrowing constraint.

The maximum amount of capital that an entrepreneur can borrow is λ times of his current asset a. Analyzing the first order conditions of the entrepreneurs' problem, one can find that the random fixed cost does not affect production decisions. However, entrepreneur's consumption, asset savings and value are decreasing in ξ, holding everything else constant. Let $c^e(a,\chi_{-1},\theta,\xi,S)$, $n^e(a,\chi_{-1},\theta,\xi,S)$, $a^e(a,\chi_{-1},\theta,\xi,S)$, $k(a,\theta,S)$, $l(a,\theta,S)$ represent the entrepreneur's optimal choices of current period consumption, labor supply, capital input, labor input and next period asset holding, respectively.

The worker's problem

Taking the real wage w and interest rates r as given, every period a worker solves the following problem

$$v^w(a,\chi_{-1},\theta,\xi,S) = \max_{\{c,n,a'\}} \left\{u(c,1-n) + \beta E[v^0(a',0,\theta',S')]\right\} \tag{8}$$

s.t.

$$c + a' \leq wn + (1+r)a - \xi\chi_{-1} \tag{9}$$

$$0 \leq n < 1, \quad a' \geq 0, \quad c > 0 \tag{10}$$

Equation (8) is the budget constraint. The worker earns labor and interest income and allocates them for consumption and saving. The idiosyncratic fixed cost ξ is paid only if the worker is an entrepreneur last period. The worker is allowed to choose labor supply. In particular, it is possible for a worker to enjoy full leisure (voluntarily unemployed). Analyzing the first order conditions of the worker's problem, one can find that consumption, asset savings and labor supply decisions depend on ξ, a, χ_{-1} and θ. Let $c^w(a,\chi_{-1},\theta,\xi,S)$, $n^w(a,\chi_{-1},\theta,\xi,S)$, $a^w(a,\chi_{-1},\theta,\xi,S)$ represent the worker's optimal choices of current period consumption, labor supply, and next period's asset holding, respectively.

Occupational Analysis

A type-θ worker (entrepreneur) with asset a chooses to switch occupation if the value as an entrepreneur (a worker) after paying the fixed cost ξ exceeds the value of continuing to be a worker (entrepreneur).

Specifically, a type-θ individual shifts occupation if he draws a fixed cost lower than the threshold ξ^T that makes him indifferent between switching and not switching, where

$$\xi^T(a,\chi_{-1},\theta,S) = \max\left\{0, \min\left\{\overline{\xi}(a,\chi_{-1},\theta,S), \xi_U\right\}\right\}$$

where the value of the fixed cost $\bar{\xi}$ depend on whether $\chi_{-1} = 0$ or 1. Specifically, $\overline{\xi}(a,0,\theta,S)$ satisfies

$$v^e(a,0,\theta,\overline{\xi};S) = v^w(a,0,\theta,0;S)^5$$

where $v^e(a,0,\theta,\overline{\xi};S)$ is the entrepreneur's value when he is a worker last period ($\chi_{-1}=0$) and pays fixed cost $\overline{\xi}$. $v^w(a,0,\theta,0;S)$ is the value when an individual continues to be a worker and pays nothing.

And $\overline{\xi}(a,1,\theta,S)$ is given by

$$v^e(a,1,\theta,0;S) = v^w(a,1,\theta,\overline{\xi};S)^6$$

where $v^w(a,1,\theta,\overline{\xi};S)$ is the worker's value when he is an entrepreneur last period (since $\chi_{-1}=1$) and pays fixed cost $\overline{\xi}$. $v^e(a,1,\theta,0;S)$ is the value when the individual continues to be an entrepreneur and pays nothing. Intuitively, given (a,χ_{-1},θ,S) an individual chooses to switch occupation whenever he draws a fixed cost smaller than the threshold $\xi^T(a,\chi_{-1},\theta,S)$, which gives him as much value as not changing his old occupation.

2.6 Equilibrium

A recursive competitive equilibrium is a set of functions: (i) real wage w and real interest rates r; (ii) value $v^0(a,\chi_{-1},\theta,S)$; (iii) Worker's allocations $c^w(a,\chi_{-1},\theta,\xi,S)$, $a^w(a,\chi_{-1},\theta,\xi,S)$ and $n^w(a,\chi_{-1},\theta,\xi,S)$; (iv) Entrepreneur's allocations $c^e(a,\chi_{-1},\theta,\xi,S)$, $a^e(a,\chi_{-1},\theta,\xi,S)$, $n^e(a,\chi_{-1},\theta,\xi,S)$, labor input $l(a,\theta,S)$ and capital allocation $k(a,\theta,S)$; (v) The evolution of household distribution over individual state variables $\mu(a,\chi_{-1},\theta)$, such that

(1) Value function $v^0(a,\theta,S)$ solves the individual's problem

(2) Allocations $c^w(a,\chi_{-1},\theta,\xi,S)$, $a^w(a,\chi_{-1},\theta,\xi,S)$ and $n^w(a,\chi_{-1},\theta,\xi,S)$ are the associated policy functions to the worker's problem.

(3) Allocations $c^e(a,\chi_{-1},\theta,\xi,S)$, $a^e(a,\chi_{-1},\theta,\xi,S)$, $n^e(a,\chi_{-1},\theta,\xi,S)$, labor input $l(a,\theta,S)$ and capital allocation $k(a,\theta,S)$ are the associated policy functions to the entrepreneur's problem.

(4) The labor market, the goods market, and the capital market all clear.[7]

(5) The evolution of household distribution over individual state variables, $\mu(a,\theta,\chi_{-1})$, is stationary (i.e., given the prices implied by this distribution, individuals' actions reproduce the same measure μ in the following period.)

[5] When $\chi_{-1}=0$, random fixed cost ξ does not influence the value of a worker as it is not paid, so $v^w(a,0,\theta,0;S) = v^w(a,0,\theta,\xi;S)$ for any $\xi \in [0,\xi_U]$.

[6] When $\chi_{-1}=1$, random fixed cost ξ does not influence the value of an entrepreneur as it is not paid, so $v^e(a,1,\theta,0;S) = v^e(a,1,\theta,\xi;S)$ for any $\xi \in [0,\xi_U]$.

[7] Please see the expressions of all market clearing conditions in the appendix.

2.7 The Labor Wedge

The labor wedge is defined and discussed in this subsection. In Chari, Kehoe and McGrattan (2007), the labor wedge τ_l is defined as the gap between the aggregate marginal product of labor (MPL) and the aggregate marginal rate of substitution (MRS). That is,

$$\tau_l \equiv log(MPL) - log(MRS) \tag{11}$$

where

$$MRS \equiv \frac{u_2(C, 1-L)}{u_1(C, 1-L)} \tag{12}$$

$$MPL \equiv (1-\alpha)\nu\frac{Y}{L} \tag{13}$$

where $\{C, L, Y\}$ are aggregate consumption, labor and output generated by the model economy, respectively.[8] Parameter ν shows up in the above equation because the production function in the economy is decreasing returns to scale.[9]

To differentiate whether the contributions for the labor wedge is from the household side or the production side, I follow Karabarbounis (2014) to assume that

$$exp(-\tau^f)MPL = w \tag{14}$$

$$exp(\tau^h)MRS = w \tag{15}$$

where τ^f denotes the component of the labor wedge due to the deviation of the aggregate MPL from the real wage and τ^h denotes the component of the labor wedge due to the deviation of the real wage from the aggregate MRS. Accordingly, the labor wedge defined in (17) equals the sum of the two components:

$$\tau_l = \tau^f + \tau^h \tag{16}$$

Thus, the labor wedge may exist either because the MPL deviates from the real wage (τ^f) or because the real wage deviates from the MRS (τ^h) or both.

To derive the labor wedge τ_l analytically, let us first obtain the first order conditions to

[8]The labor wedge is measured using the model aggregates $\{C, L, Y\}$ as if they are the aggregate consumption, labor and output that are generated by an economy with a representative household and a representative production sector. Specifically, the representative household has the utility function $u(C, 1-L) = \frac{\left(C^{\tau}(1-L)^{(1-\tau)}\right)^{1-\sigma}-1}{1-\sigma}$, and the representative production sector has the production technology $Y = zK^{\alpha\nu}L^{(1-\alpha)\nu}$.

[9]The production scale parameter $\nu = 1$ in CKM (2007).

the entrepreneur's problem.[10] For all $i \in [0,1]$, we have

$$[c_i] \quad u_1(c_i, 1-\bar{n}) = \phi_{1i} \tag{17}$$

$$[k_i] \quad \alpha\nu z\theta_i k_i^{\alpha\nu-1}(\bar{n}+l_i)^{(1-\alpha)\nu} = (r+\delta) + \frac{\phi_{2i}}{\phi_{1i}} \tag{18}$$

$$[l_i] \quad (1-\alpha)\nu z\theta_i k_i^{\alpha\nu}(\bar{n}+l_i)^{(1-\alpha)\nu-1} = w - \frac{\phi_{3i}}{\phi_{1i}} \tag{19}$$

where ϕ_{1i}, ϕ_{2i} and ϕ_{3i} are the Lagrange multipliers to the budget constraint equation (5), the collateral borrowing constraint equation (6) and the condition $l_i \geq 0$ for labor demand, respectively.

According to condition (19), all entrepreneurs can be divided into two subsets. The first subset contains entrepreneurs who hire strictly positive amount of labor from the market, i.e., $l_i > 0$. Each entrepreneur in this subset has his marginal product of labor equal to the wage rate, i.e., $\phi_{3i} = 0$ and $mpl_i \equiv (1-\alpha)\nu\theta z k_i^{\alpha\nu}(\bar{n}+l_i)^{(1-\alpha)\nu-1} = w$. The second subset contains entrepreneurs that hire no labor from the market $l_i = 0$. In this case, the marginal product of labor is smaller than the wage, i.e., $\phi_{2i} > 0$ and $mpl_i < w$.

For the convenience of analysis, let $\mathbf{E} \equiv \{i \in [0,1] | v_i^e(a, \chi_{-1}, \theta, \xi, S) \geq v_i^w(a, \chi_{-1}, \theta, \xi, S)\}$ denote the set of all entrepreneurs in the economy, $\mathbf{E}_1 \equiv \{i \in \mathbf{E} | l_i > 0\}$ denote the subset of entrepreneurs that hire labor from the market and $\mathbf{E}_2 \equiv \{i \in \mathbf{E} | l_i = 0\}$ denote the subset of entrepreneurs that do not hire any workers other than themselves.

With these notations, the aggregate marginal product of labor is:

$$\begin{aligned} MPL &= (1-\alpha)\nu\frac{Y}{L} \\ &= \frac{(1-\alpha)\nu\left[\int_{i\in E_1} y_i di + \int_{i\in E_2} y_i di\right]}{\int_{i\in E_1}(\bar{n}+l_i)di + \int_{i\in E_2}(\bar{n}+l_i)di} \qquad (20) \\ &\leq w \qquad (21) \end{aligned}$$

The inequality "$\leq$" shows up because for all $i \in E_1$ we have $\frac{(1-\alpha)\nu y_i}{\bar{n}+l_i} = w$ and for all $i \in E_2$ we have $\frac{(1-\alpha)\nu y_i}{\bar{n}+l_i} < w$. The equality "$=$" holds if and only if the measure of entrepreneurs that hire no labor is zero, i.e. $\gamma(E_2) \equiv \int_{i\in E_2} di = 0$.[11] In this case, the MPL is equal to the real wage, $MPL = w$. Otherwise, we have $MPL < w$.

Now let us derive the MRS. Since entrepreneurs must work a fixed amount of time $\bar{n} > 0$, they do not have the optimality conditions for the intratemporal choices between consump-

[10] The individual index i is written explicitly here to avoid confusion, $i \in [0,1]$.

[11] Let Σ be the σ-algebra over $[0,1]$. Then $([0,1],\Sigma)$ is a measurable space and γ: $\Sigma \to [0,1]$ is a positive measure over $([0,1],\Sigma)$. It is easy to verify that E, E_1 and E_2 are measurable sets in Σ.

tion and leisure (i.e. the labor-leisure condition). Nevertheless, entrepreneurs' labor is held fixed for two reasons. First, Bils, Klenow and Malin (2015) find that the elasticity of annual hours with respect to real annual GDP in the U.S. data are relatively more cyclical for workers than for self-employed people. Second, fixing entrepreneurs' hours can isolate the impact of a credit crunch and entry and exit of production on the variation of the labor wedge.

Workers can choose their labor supply and the labor-leisure condition for worker i is

$$\frac{u_2(c_i, 1-n_i)}{u_1(c_i, 1-n_i)} = w + \frac{\varphi_{2i}}{\varphi_{1i}} \tag{22}$$

where φ_{1i} and φ_{2i} are the Lagrange multipliers for worker i's budget constraint equation (9) and the condition $n_i \geq 0$, respectively.

Equation (22) implies that workers can be divided into two subsets. The first subset are workers who choose to work, $n_i > 0$, and thus each worker in this subset has his marginal rate of substitution between consumption and leisure equal to the wage, i.e., $mrs_i \equiv \frac{(1-\tau)c_i}{\tau(1-n_i)} = w$. The second subset includes workers who choose to enjoy full leisure so that $n_i = 0$. We can regard them as voluntarily unemployed. For each worker in the second group, his marginal rate of substitution between consumption and leisure is larger than the wage, i.e., $mrs_i > w$. Since the workers' labor supply in the economy is decreasing in asset holdings,[12] a small fraction of workers with very high assets may choose to supply zero labor.

For the convenience of analysis, let me denote $\mathbf{H} \equiv \{i \in [0,1] | v_i^e(a, \theta, \chi_{-1}, S) < v_i^w(a, \theta, \chi_{-1}, S)\}$ as the set of all workers in the economy, $\mathbf{H}_1 \equiv \{i \in \mathbf{H} | n_i > 0\}$ as the subset of workers with strictly positive labor supply and $\mathbf{H}_2 \equiv \{i \in \mathbf{H} | n_i = 0\}$ as the subset of voluntarily unemployed workers.

If the real wage w does not always equal to the marginal rate of substitution at the individual level, will w follow to deviate from the aggregate marginal rate of substitution (MRS)? The answer is that it depends. Using some algebra I can obtain:[13]

$$MRS \equiv \frac{u_2(C, 1-L)}{u_1(C, 1-L)} \tag{23}$$

$$= \frac{1-\tau}{\tau} \frac{C}{1-L} \tag{24}$$

$$= w + \frac{\eta_1 + \eta_2}{1-L} \tag{25}$$

where

$$\eta_1 \equiv \frac{1-\tau}{\tau} C_e - w(1-\bar{n})\gamma_e$$

[12] See Figure A.3 and the discussion about it in Section 4.
[13] See appendix for the derivation of MRS.

$$\eta_2 \equiv \frac{1-\tau}{\tau} C_u - w\gamma_u$$

$$C_e \equiv \int_{i\in\mathbf{E}} c_i di, \quad C_u \equiv \int_{i\in\mathbf{H}_2} c_i di, \quad \gamma_e \equiv \int_{i\in\mathbf{E}} di, \quad \gamma_u \equiv \int_{i\in\mathbf{H}_2} di$$

where C_e is entrepreneurs' aggregate consumption, C_u is the aggregate consumption of voluntarily unemployed workers, γ_e is the measure of all entrepreneurs, and γ_u is the measure of voluntarily unemployed workers. How can we understand η_1 and η_2 economically? Notice that the aggregate marginal rate of substitution for all entrepreneurs (MRS_e) is:

$$MRS_e = \frac{u_2(C_e, \gamma_e(1-\bar{n}))}{u_1(C_e, \gamma_e(1-\bar{n}))} = \frac{1-\tau}{\tau} \frac{C_e}{(1-\bar{n})\gamma_e} \tag{26}$$

If the MRS_e is equal to the real wage, then η_1 equals zero. Thus η_1 measures the deviation of the real wage from the MRS_e. Similarly, η_2 measures the deviation of the real wage from the aggregate marginal rate of substitution for all voluntarily unemployed workers (MRS_u).

Using the analytic expressions for the MRS and the definition of the deviation of the real wage from the MRS, i.e. τ^h, in condition (15), we can obtain

$$\tau^h = -(d_1 + d_2) \tag{27}$$

where

$$d_1 \equiv \frac{\eta_1}{w(1-L)} \quad \text{and} \quad d_2 \equiv \frac{\eta_2}{w(1-L)} \tag{28}$$

Thus d_1 equals the deviation of the real wage from the MRS_e divided by the aggregate opportunity cost of leisure, $w(1-L)$. And d_2 equals the deviation of the real wage from the MRS_u divided by the aggregate opportunity cost of leisure. Moreover, equation (27) shows that the deviation of real wage w from the MRS depends on d_1 and d_2. Let us call d_1 the entrepreneurial deviation and d_2 the workers' deviation. Accordingly, the gap between the real wage and the MRS is consisted of two components: an entrepreneurs' deviation d_1 and a workers' deviation d_2.

The labor wedge is discussed in the following two propositions:

Proposition 1 *If the measure of entrepreneurs that hire no labor from the market is zero in the economy, i.e. $\gamma(E_2) = 0$, then we have $MPL = w$ and the labor wedge equals to the deviation of the real wage from the MRS: $\tau_l = \tau^h = -(d_1 + d_2)$.*[14]

[14] The result here does not conflict with the standard representative-agent model (or models with homogeneous agents) as the standard representative-agent model without labor market distortions is a special case with $\gamma(E_2) = \gamma(H_2) = 0$ and $d_1 = d_2 = 0$. Thus the labor wedge $\tau_l = 0$ in the standard neoclassical model with representative agents.

If in the economy the measure of entrepreneurs that hire no labor from the market is zero, then the equality "=" in condition (21) would hold and we have $MPL = w$. In this case, there is no gap between the MPL and the real wage. Then using the definition of the labor wedge in equation (17), the results of the labor wedge in Proposition 1 is obvious.

Proposition 2 *If in the economy there is strictly positive measure of entrepreneurs that do not hire labor from the market, i.e.* $\gamma(E_2) > 0$*, then we have* $MPL < w$ *and the labor wedge*

$$\begin{cases} \tau_l < 0 & \text{if } d_1 + d_2 \geq 0 \text{, so } MRS \geq w \\ \tau_l \quad is \quad indeterminate & \text{if } d_1 + d_2 < 0 \text{, so } MRS < w. \end{cases}$$

The proof of Proposition 2 is shown in the appendix.

Proposition 1 and 2 imply that the labor wedge can arise endogenously in the economy without labor market distortions through two channels or both. Proposition 1 represents the first channel. The labor wedge would arise when individual heterogeneity is such that the real wage deviates from the MRS between consumption and leisure. In this case, the deviation can originate from two sources: (1) there is a gap between the entrepreneurs' aggregate marginal rate of substitution and the real wage so that $d_1 \neq 0$; (2) a positive measure of workers choose to enjoy full leisure so that their aggregate marginal rate of substitution is greater than the real wage and $d_2 > 0$.

The second channel for the labor wedge to exist is when the heterogeneity in the economy is such that the MPL is not equal to the real wage. This is the case when a positive measure of entrepreneurs in the economy do not hire labor from the labor market. In this case, the labor wedge arises endogenously in the economy even if the real wage may or may not deviate from the MRS at the same time. Proposition 2 shows the second channel, even if the first channel may or may not present simultaneously.

Propositions 1 and 2 imply that an economy would not have the labor wedge only if two conditions can be satisfied simultaneously: (a) the economic heterogeneity is such that all entrepreneurs hire labor so that the MPL has no deviation from the real wage, and (b) the interactions between entrepreneurs and voluntarily unemployed workers is such that the real wage equals the MRS. A standard representative-agent model in which the representative household selects labor supply and the representative final good firm with capital and labor as inputs satisfies condition (a) because the representative firm hires labor to produce. It also satisfies condition (b) because the representative household's optimal consumption and labor supply are such that the marginal rate of substitution between consumption and leisure is equal to the real wage. Therefore, a standard neoclassical representative agent model cannot generate the labor wedge.

3 Parametrization

One period in the model is a year. This empirical counterpart of an entrepreneur in the model is an establishment in the U.S. data. I choose $\sigma = 1.5$ following standard practice. I set $\delta = 0.06$ to have an annual capital depreciation rate of 6% in the steady state. Assume the aggregate productivity follows a mean zero *log* $AR(1)$ process

$$logz' = \rho_z logz + \epsilon_z, \quad \epsilon_z \sim N(0, \sigma^2_{\epsilon_z}) \tag{29}$$

The persistence of productivity ρ_z is 0.859 and the standard deviation of disturbance σ_{ϵ_z} is 0.014 following Khan and Thomas (2013). Entrepreneurial ability θ evolves according to the following *log AR(1)* process

$$log\theta_{t+1} = \rho_e log\theta_t + \epsilon_{e,t}, \quad \epsilon_e \sim N((1-\rho_e)log\left(\mu_e\right), \sigma^2_{\epsilon_e}) \tag{30}$$

The three parameters μ_e, ρ_e, σ_{ϵ_e} are pinned down together by targeting three moments in the data: the establishment exit rate, Gini coefficient for wealth and top 10% employment. According to the U.S. Census Business Dynamics Statistics, the annual establishment exit rate from 1977 to 2012 is 10.5%. The persistence parameter ρ_e is 0.8 to hit this target. The decile with the largest firms in the Business Dynamic Statistics accounts for 78 percent of the total employment from 2007 to 2012, the standard deviation of the disturbance term σ_{ϵ_e} equals 0.28 so that the top 10 percent employment share is about 0.78.[15] Diaz-Gimenez et al (1997) reported that the Gini coefficient for wealth is 0.78 in the 1992 SCF. The parameter $\mu_e = 0.45$ so that the Gini coefficient for wealth is 0.68 in the benchmark economy which is close to the target and stands among other papers, such as Bassetto, Cagetti and De Nardi (2015).

The rest of the parameters are calibrated together to hit six data moments. Discount factor $\beta = 0.95$ so that the steady state annual interest rate is 4.0%. Parameter $\tau = 0.38$ so that individuals in the economy on average work one third of the time. I adopt $\alpha = 0.30$ to generate a capital-output ratio of about 2.60 in the steady state. According to the Survey of Consumer Finance for the year 2001 and 2004, the net worth of the richest 10 percent

[15]The Business Dynamic Statistics does not publicly disclose details about firm size distribution before 2007. So I use the average of the top 10% employment from 2007 to 2012 as the target moment.

	Target moment	Model	Target	Source
β	Real interest rate	4.0	4.0	U.S. data
τ	Average hours worked	0.33	0.33	U.S. data
α	Capital-output ratio	2.57	2.40	U.S. data
ν	Top 10 percent net worth share	0.57	0.55	SCF (2001 & 04)
λ	External finance to GDP	1.70	1.75	Buera and Shin (2013)
ξ_U	Percentage of entrepreneurs	12.0	11.7	SCF(2001 & 04)
ρ_e	Percentage of exiting entrepreneurs	10.0	10.5	BDS (1977-2012)
σ_e	Top 10 percent employment share	0.77	0.78	BDS (2007-12)
μ_e	Gini coefficient for wealth	0.68	0.78	Diaz-Gimenez et al (1997)

Table 1: Model and Data generated moments

Data source: NIPA tables, Business Dynamics Statistics (BDS) and the Survey of Consumer Finance (SCF).

accounts for 55 percent of the total net worth in the economy. Scale parameter ν equals 0.89 to target the top 10% net worth share. External finance in the model is the borrowed capital that cannot be covered by entrepreneurs' private assets (or internal funds).[16] The tightness of financial constraints λ is equal to 8.5 to hit an external finance to GDP ratio close to 1.75 as in the U.S. data. According to the Survey of Consumer Finance, entrepreneurs take up 11.7% of total population from 2001 to 2007. The upper bound of the fixed cost process ξ_U equals 0.35 to hit the entrepreneurial population target. As shown in Table 1, the model generated moments well match the corresponding moments in the U.S. data.

4 Steady State

In this section I describe the steady state properties of the model economy. Figure A.2 shows the expected discounted lifetime value function $v^0(a, \theta, \chi_{-1}, S)$. Firstly, given ability θ and occupational history χ_{-1}, an individual's expected lifetime value is increasing in his current asset a. Secondly, holding asset a and history χ_{-1} constant, a person's expected value is increasing in his ability. Finally, holding asset and ability constant, a high-ability individual has higher value when he is an entrepreneur last period. In contrast, a median-ability or low-ability individual has higher value when he is a worker last period. It is worth noticing that high-ability individuals with very small assets have higher value as workers. When asset

[16] The definition of external finance in the model is $EF = \int_{i \in \mathbf{E}} (k_i - a_i) di$ where $\mathbf{E}$ denotes the set of entrepreneurs.

holding is very small, the production profit is tiny even for high ability individuals. In this case, being workers and earn labor income gives them larger value as they can accumulate assets and enter the production sector when asset holdings are larger.

Holding ability and asset constant, differences in value caused by occupational switch are quite small because the fixed cost for changing occupation is only a one-time payment and thus has very small impact over any individual's lifetime value.

The collateral borrowing constraint gives constrained individuals additional incentives to save besides earning interest income.[17] Specifically, an individual tends to save more if two conditions are satisfied simultaneously: (i) The individual is constrained in the financial markets. (ii) The individual has strictly positive probability to become an entrepreneur. Intuitively, assets have two sources of returns. One is interest income. The other is the increase in production profit. When an entrepreneur is constrained in the financial market, increasing one unit of asset allows him to increase λ units of capital input. Thus production profit is increasing in asset.

Nevertheless, an individual does not have the second return in saving if he cannot satisfy conditions (i) and (ii) at the same time. For example, suppose condition (i) fails so that the individual is an unconstrained entrepreneur. In this case, saving an additional unit of asset this period does not increase his future capital input because his borrowing constraint is not binding. Thus his only return from saving is the interest income.

The right panel in Figure A.4 shows the asset decision rules of an entrepreneur. There are two important observations from it. Firstly, given θ, χ_{-1} and ξ, future asset a' is increasing in the entrepreneur's current asset. Also, given a, χ_{-1} and ξ, an entrepreneur with higher θ saves more asset for the next period. The reason is because production profit is increasing in a and θ. Second, a high-ability entrepreneur's asset decision rule is concave in asset a, while a low-ability entrepreneur's asset decision rule is first flat and then linearly increasing in a. As shown by Figure A.5, with the parameters identified in the benchmark economy, a high ability entrepreneur with asset holdings less than 2.6 units of consumption good is constrained in the financial market and production profit is concave in a.[18] Thus the constrained entrepreneur's income and saving are also concave in a. A low-ability entrepreneur's asset decision rule is first flat because he has little profit with limited capital input. In this case, he chooses not to save in order to have consumption.[19]

[17] See my online appendix for mathematical proof.

[18] For constrained entrepreneurs, profit function is concave in a because $\frac{\partial^2 \pi(a,\theta,z,\lambda)}{\partial a^2} = (\alpha\nu - 1)\alpha\nu z\theta\lambda^{\alpha\nu} a^{\alpha\nu-2} (\bar{n})^{(1-\alpha)\nu}$ if $l = 0$ and $\frac{\partial^2 \pi(a,\theta,z,\lambda)}{\partial a^2} = \frac{(\nu-1)\alpha\nu}{(1-(1-\alpha)\nu)^2} (z\theta)^{\frac{1}{1-(1-\alpha)\nu}} \left(\frac{(1-\alpha)\nu}{w}\right)^{\frac{(1-\alpha)\nu}{1-(1-\alpha)\nu}} \lambda^{\frac{\alpha\nu}{1-(1-\alpha)\nu}} a^{\frac{\alpha\nu}{1-(1-\alpha)\nu}-2} [1 - (1-\alpha)\nu]$ if $l > 0$. In both cases we have $\frac{\partial^2 \pi(a,\theta,z,\lambda)}{\partial a^2} < 0$. See the appendix for more details.

[19] In the steady state, no low ability and assets individuals choose to be entrepreneurs as they have higher value to be wage earners.

The left panel of Figure A.4 shows the asset decision rule of a worker and four observations can be raised with respect to it. First, given a worker's ability, the optimal asset holding is increasing in current asset. Second, a worker's asset decision curves are first convex and then linear in asset a. This is because low-asset workers satisfy conditions (i) and (ii) simultaneously and thus have more incentive to save. Third, high-ability workers with high assets save less than low- and median-ability workers because they have higher expected lifetime income which makes them consume more. In contrast, high-ability workers with small assets save more than low- and median-ability workers out of precautionary purpose. Since individual ability is persistent, more assets enable them to borrow more capital when they become entrepreneurs. Finally, workers' asset decisions do not cross the zero point. Although ability is persistent, a low-ability worker still has a small chance to become high-ability next period. In that case, positive asset holding enables him to borrow capital and carry out production.

Figure A.6 displays the invariant distribution of individuals in the steady state. The top graph is the distribution of households that are workers. First, most of the median- and low-ability individuals are located in this graph. In principle, it is optimal for low- and median-ability individuals to become workers as they are less efficient in managing production. Second, low-ability individuals are crowded at an asset level above zero for precautionary savings motive: positive asset holdings enable them to borrow capital and carry out production in case they become entrepreneurs. The bottom graph in Figure A.6 shows the distribution of entrepreneurs. In equilibrium, 99.8% of high-ability individuals are entrepreneurs and 59% of entrepreneurs hold assets less than 4.9 units of consumption good. Thus most production sizes are small in terms of employment in the steady state.[20]

With the parameters identified in calibration, the labor wedge arises endogenously in the steady state. The aggregate marginal product of labor is approximately equal to the real wage as only 0.06% of entrepreneurs do not hire labor other than themselves.[21] Therefore, the labor wedge arises mainly through the channel in Proposition 1: the real wage deviates from the aggregate marginal rate of substitution between consumption and leisure. On the one hand, workers with very high assets choose to be unemployed voluntarily. Consequently, their marginal rate of substitution is larger than the wage. In the steady state, 1.73% of workers belongs to this group so that $d_2 > 0$. On the other hand, entrepreneurs work fixed hours and it turns out their MRS_e in condition (26) does not equal to the real wage and hence $d_1 > 0$. Because of these two deviations, the MRS does not equal to the MPL. With the parameters identified in the calibration, the labor wedge is -0.0744 in steady state.

[20] Recall that entrepreneurs' labor hiring decisions are increasing in asset holdings as shown in Figure A.3.

[21] Recall that entrepreneurs take up 12% of the total population in steady state.

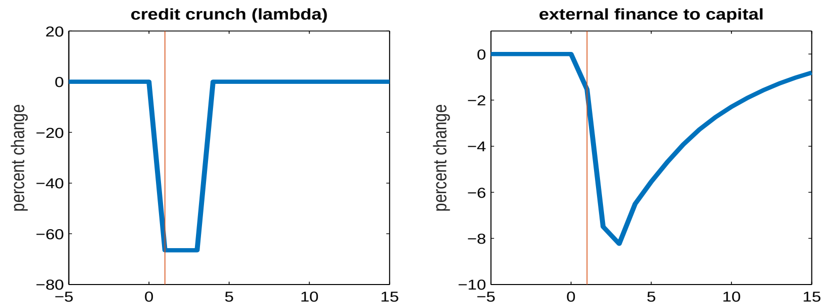

Figure 2: Credit Crunch

Note.– The credit crunch is a 63% decline in λ for three consecutive years. The size of the decline is chosen to target the decline of external finance to capital over the Great Recession.

5 The Credit Crunch

I consider the aggregate dynamics of the model following an unexpected tightening of the collateral borrowing constraints by reducing the coefficient λ.[22] Buera, Fattal Jaef and Shin (2014) reports an 8% decline in the stock of credit market liabilities to non-financial assets of the non-financial business sector in the US from the fourth quarter of 2008 to the first quarter of 2010. I follow them to pin down the size of the credit crunch by targeting this ratio. Shown in Figure 2, the λ series are chosen to generate a 8% decline in the ratio of external finance to capital stock in the model to match the magnitude observed in the data.

As shown in Figure 3, the credit crunch causes a persistent contraction in real economic activity (solid blue lines). Output experiences a sharp decline as the tighter collateral constraint reduces capital and labor inputs for constrained entrepreneurs and causes losses in TFP. The reduction in endogenous TFP is achieved via resource reallocation in the economy. First, the tighter borrowing constraints make capital allocation more distorted in both the extensive margin and intensive margin. For one thing, a larger fraction of entrepreneurs becomes constrained in the financial markets with smaller λ. Output falls as more entrepreneurs are producing with lower capital inputs. For another, entrepreneurs who are constrained in the steady state are constrained by more as the collateral constraint becomes tighter. In contrast, unconstrained entrepreneurs produce with higher capital and labor as factor prices fall upon the arrival of the shock. Second, the entry-and-exit dynamics in the production sector crowds out a tiny fraction of high-ability constrained entrepreneurs. Productivity falls as entrepreneurs are less productive on average. A growing literature have found that credit crunches can cause losses of aggregate productivity. Leading examples are Gilchrist, Sim

[22] The credit crunch hits unexpectedly and individuals have perfect foresight over the path of λ once the shock takes place.

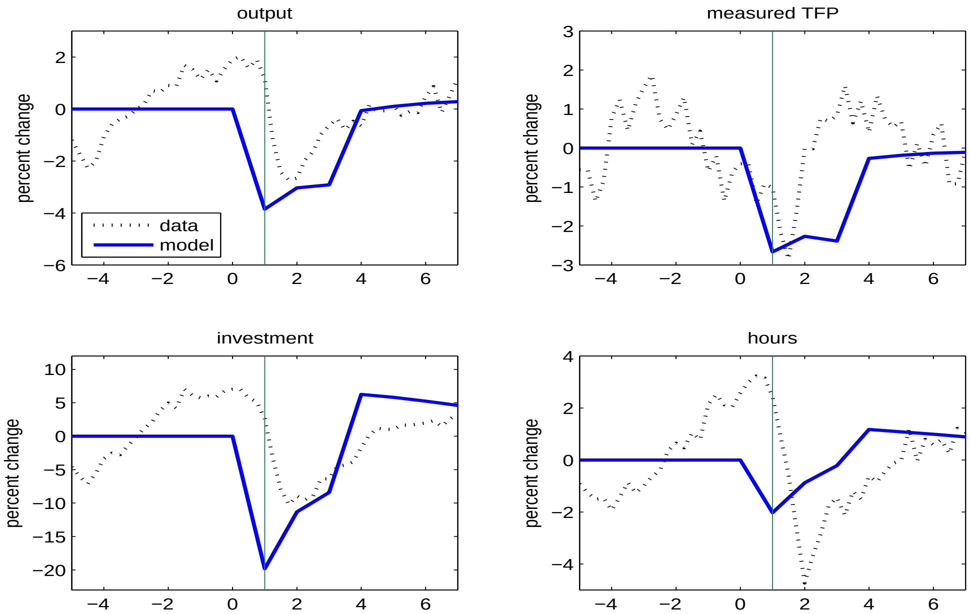

Figure 3: Credit Crunch

Note.– The blue curve is for the benchmark credit crunch. The dashed black curve is the U.S. data. Data variables are real and HP-filtered with smoothing parameter 1600. Shock arrives unexpectedly in Year 1 which corresponds to 2008.

and Zakrajsek (2014), Khan and Thomas (2013), and Buera and Moll (2015).

The labor wedge experiences significant movements in the credit crunch as shown in Figure 4 (solid blue line, upper right panel). The labor wedge τ_l declines slightly in the first year and then rises up 9.2% in period 3. After that it declines gradually. Since the measure of entrepreneurs that do not hire labor from the market is only 0.2% of all entrepreneurs, the MPL is approximately equal to the real wage w. Thus Proposition 1 is valid in this experiment and the labor wedge is attributable to the deviation of the real wage from the MRS.[23] That is,

$$\tau_l = \tau^h = -(d_1 + d_2) \tag{31}$$

where

$$d_1 \equiv \frac{\eta_1}{w(1-L)} \quad \text{and} \quad \eta_1 \equiv \frac{1-\tau}{\tau} C_e - w(1-\bar{n})\gamma_e \tag{32}$$

$$d_2 \equiv \frac{\eta_2}{w(1-L)} \quad \text{and} \quad \eta_2 \equiv \frac{1-\tau}{\tau} C_u - w\gamma_u \tag{33}$$

[23] $MPL = 0.99999 * w$ in the steady state while it becomes $0.99997 * w_t$ in years 1 to 3. Although this deviation increases in the credit crunch, it is definitely negligible. In contrast, $MRS = 1.0744 * w$ in the steady state while it becomes $1.0697 * w_1$, $1.0740 * w_2$, $1.0790 * w_3$ in years 1, 2, 3 respectively. Therefore, the deviation of the MRS from the wage rate is the dominating factor in generating the labor wedge.

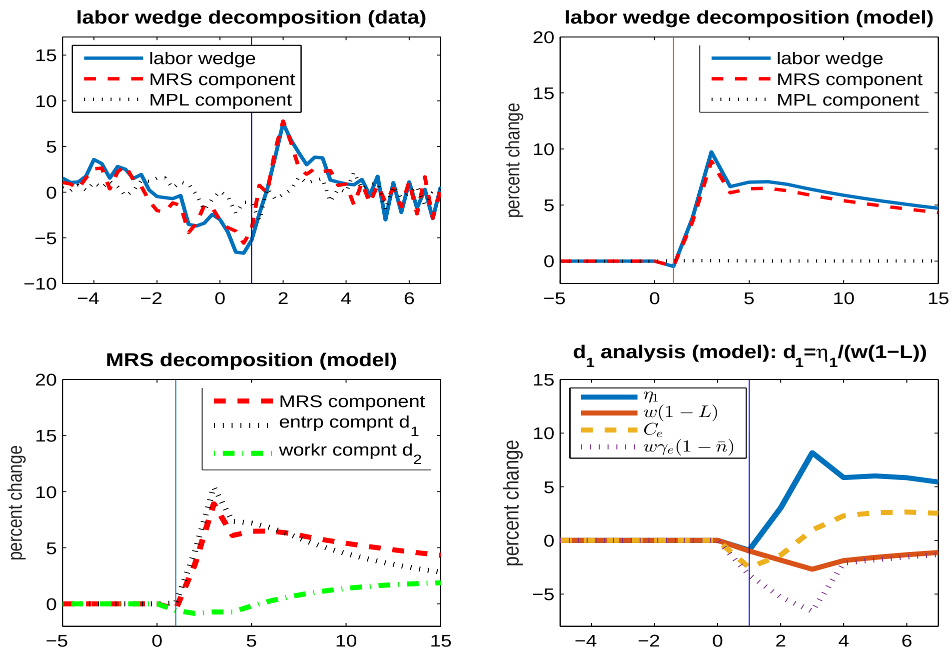

Figure 4: Decomposition of the Labor Wedge in the Data and the Credit Crunch

Note.– The Shock arrives unexpectedly in year 1 which corresponds to the year 2008. Data variables are HP-filtered with smoothing parameter 1600. In the bottom right panel, the expression for d_1, η_1 is in equation (32).

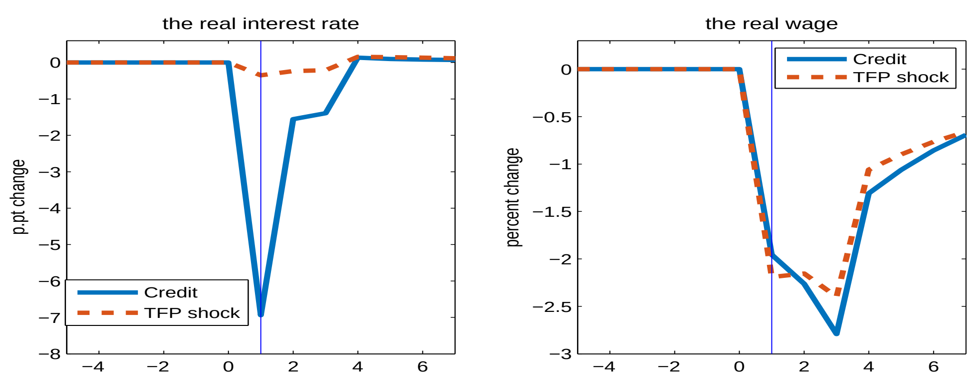

Figure 5: Factor Prices in the Credit Crunch and the TFP shock

Note.– Blue curve is for the credit crunch. Dashed orange curve is for the TFP shock. The real interest rate is percent point change. The real wage are percent change from steady state. Shock arrives unexpectedly in Year 1 which corresponds to 2008.

According to the decomposition of the labor wedge in Figure 4 (upper right panel), the labor wedge increases in the credit crunch mainly because the real wage has larger deviation from the MRS — the sum of two deviations $d_1 + d_2$ increases. First, the entrepreneurs' deviation d_1 plays a predominant role. In the year of the shock, entrepreneurs experience significant declines in production profits, so they reduce their consumption C_e (Fig. 4 bottom right panel). Meanwhile, the entrepreneurs' opportunity cost of leisure, $w(1-\bar{n})\gamma_e$, decline by a similar size. The reductions in entrepreneurs' opportunity cost of leisure and consumption make d_1 decline a bit according to equation (32). From year 2 and on, the opportunity cost of leisure continues declining while entrepreneurs' consumption starts to recover. As a result, the gap between the real wage and the MRS_e —measured by η_1— is larger since year 2 and thus d_1 increases significantly. Second, the workers' deviation d_2 shows indistinguishable movements. Although the measure of voluntarily unemployed workers increase 3.37% upon the arrival of the shock, their consumption and opportunity cost of leisure both decline. It turns out that the workers' deviation only decreases by a small margin. The dynamics in entrepreneurs' deviation d_1 and workers' deviation d_2 together generate significant increases of the labor wedge.

When the borrowing condition starts to restore in year 4, the deviation of the aggregate MRS from the real wage becomes smaller and the labor wedge gradually recovers to the steady state level.

5.1 Comparing the credit crunch with exogenous TFP shock

The previous analysis shows that credit crunches could bring damages to the real economy and increase the variation of labor wedge. My goal in this section is to compare the impact of a credit crunch to that of a negative TFP shock, which is a more standard source of business cycle fluctuations. To implement this experiment, the economy starts in its stationary equilibrium and is hit with an unanticipated drop and recovery in aggregate TFP, z_t in equation (29). Following Buera et al (2014), this exogenous TFP series is constructed to mimic the endogenous TFP dynamics from the credit crunch.

Figure 6 depicts the dynamics of aggregate quantities in response to the exogenous aggregate TFP shock (solid blue lines). It also shows the dynamics in response to the credit crunch (dashed red curves) for comparison. Despite the identical TFP dynamics of the two experiments, the credit crunch exercise shows slightly sharper contractions in output and investment. Moreover, the initial decline in hours is smaller in the TFP shock.

The labor wedge arises in the TFP shock, but the magnitude of its increment is smaller

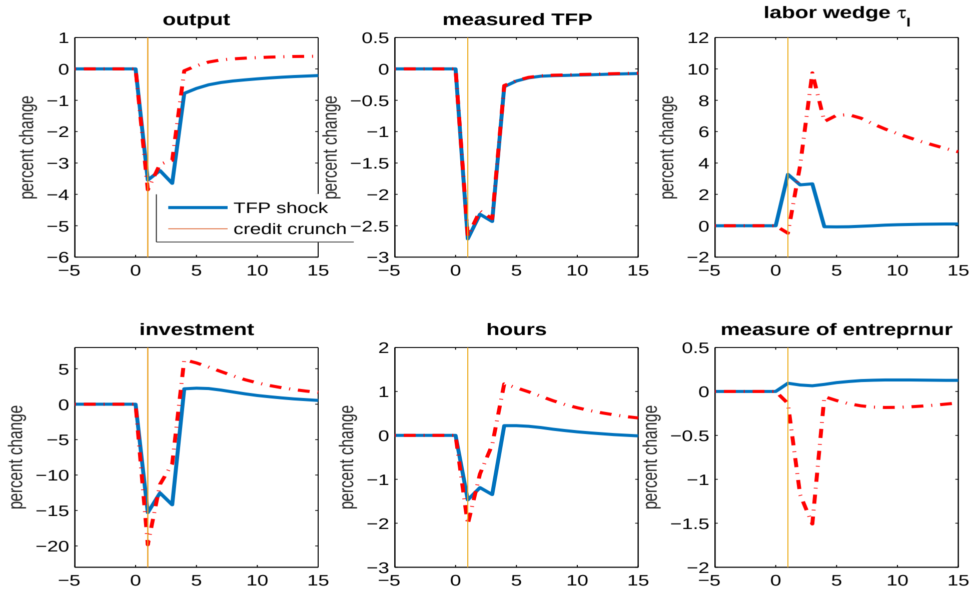

Figure 6: the Credit Crunch and the exogenous TFP shock

Note.– The shock arrives unexpectedly in year 1 which corresponds to the year 2008. All variable are percent change from steady state.

than that in the credit crunch except the first year of shock.[24] Since the measure of entrepreneurs that do not hire labor is tiny, the MPL is equal to the real wage and Proposition 1 holds in the TFP shock. Therefore, equations (31)-(33) are valid and the labor wedge increases in the TFP shock mainly due to the deviation of the real wage from the MRS.

The analysis of the labor wedge is shown in Figure 7. Comparing to the credit crunch exercise, the labor wedge exhibits smaller increases in the TFP shock because the deviation of the real wage from the MRS—measured by $(-(d_1 + d_2))$— is smaller. First, the workers' term d_2 has indistinguishable variations in both experiments. Second, d_1 has larger increases in the credit crunch due to the opposite movements of entrepreneurs' consumption and opportunity cost of leisure, which does not happen in the TFP shock. In the credit crunch, entrepreneurs' consumption C_e starts to recover rapidly from year 2 of the shock while their opportunity cost of leisure $w(1 - \bar{n})\gamma_e$ keeps declining (Fig. 7 bottom right panel). By equation (32), entrepreneurs' deviation d_1 increases significantly from year 2 of the shock. In the TFP shock, however, both entrepreneurs' consumption and opportunity cost of leisure tend to decrease slightly in year 2. Accordingly, d_1 has smaller variation and the labor wedge is smaller in the TFP shock.

[24]Since we are more interested in the increase of labor wedge in the credit crunch, I focus on explaining the years that labor wedge has significant movements. In year 1 of shock, the entrepreneurial component d_1 is smaller in the credit crunch because entrepreneurial good consumption C_e has sharp decline so that the numerator η_1 in d_1 is smaller.

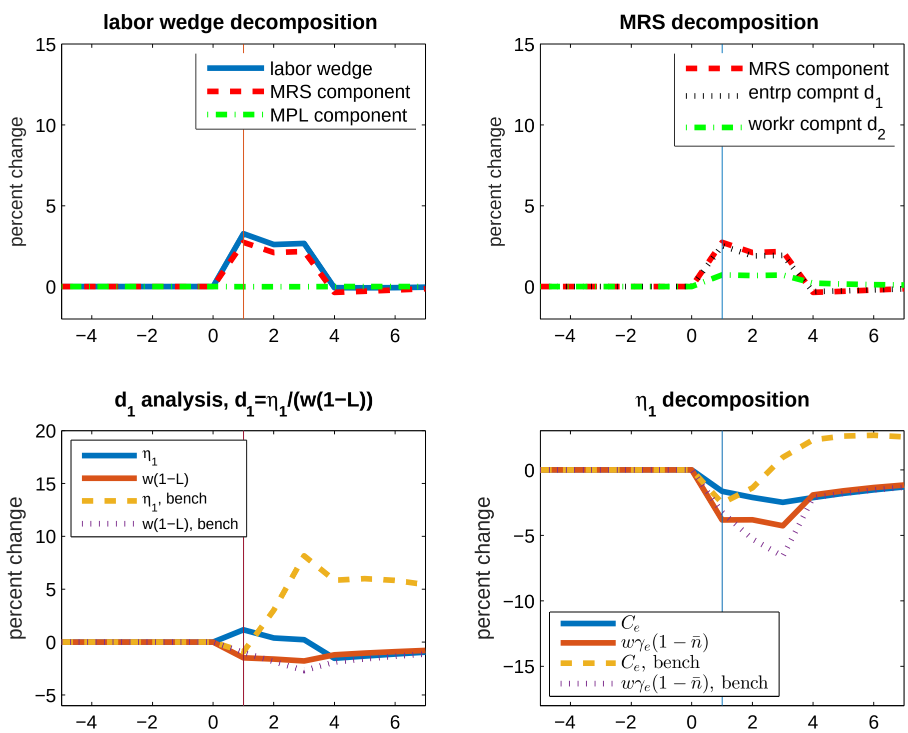

Figure 7: The Analysis of Labor Wedge in the TFP shock

Note.– The shock arrives unexpectedly in year 1 which corresponds to the year 2008.

It is critical that entrepreneurs' consumption C_e recovers differently in the credit crunch and the TFP shock. Intuitively, smaller TFP in the exogenous TFP shock causes reductions in profits and consumption for all entrepreneurs symmetrically regardless of their financial positions. In the credit crunch, however, constrained entrepreneurs experience sharp declines in profits and consumption while unconstrained entrepreneurs expand production scale to increase profits and consumption as factor prices are lower.[25] In the mean time, constrained entrepreneurs with low profits choose to exit production. The presence of higher consumption unconstrained entrepreneurs and exits of low consumption constrained entrepreneurs make the entrepreneurs' consumption C_e to rebound rapidly since year 2 of the credit crunch. In contrast, the recovery of C_e in the exogenous TFP shock starts from year 4 and its strength is much weaker.

The core mechanism for the differential responses of entrepreneurs' opportunity cost of leisure, $w(1 - \bar{n})\gamma_e$, is that the tighter collateral constraints greatly reduce capital input for constrained entrepreneurs in the credit crunch, but this mechanism does not exist in the negative TFP shock. Figure 6 (bottom right panel) shows two separate effects of this mechanism. First, capital input is limited by the tighter collateral constraints so that the MPL (which equals to w) is lower in the credit crunch. Second, lower capital input reduces profits and forces a fraction of constrained entrepreneurs to exit production. As a result, the measure of entrepreneurs decline significantly. In the exogenous TFP shock, however, the measure of entrepreneurs γ_e show little variation instead. These two separate effects together lead to continuous decline of the entrepreneurs' opportunity cost of leisure, $w(1 - \bar{n})\gamma_e$, in the credit crunch until the collateral constraints start to recover.

Notice that the fixed hours $\bar{n}$ for entrepreneurs does not cause fluctuations of the labor wedge. Although $\bar{n}$ may account for the existence of entrepreneurial deviation d_1 in the steady state, $\bar{n}$ itself does not generate fluctuations of the labor wedge over the business cycle. As we have discussed previously, the variation of entrepreneurs' deviation d_1 in different shocks are caused by the differential responses in entrepreneurial consumption, measure of entrepreneurs and the real wage, rather than by the fixed hours $\bar{n}$.

Moreover, Bils, Klenow and Malin (2015) compare the labor wedge of the self-employed people to the labor wedge for the whole economy using the U.S. data from 1987 to 2012. Although the fluctuations of the labor wedge over the business cycle do not all originate from self-employed people in the U.S. data, Bils et al (2015) have showed that the labor wedge of the self-employed people is suggestive of a countercyclical labor wedge for the overall economy. This finding could serve as an empirical support for my analysis.

[25] Higher profits for unconstrained entrepreneurs attract a small fraction of rich workers to enter production, but this effect is tiny.

5.2 The role of endogenous entry and exit of production

The number of U.S. establishments declined about 2.8% from 2007 to 2009 as a large number of establishments exited the production sector. As analyzed in the previous section, the entrepreneurial entry and exit dynamics is relevant for the movements of labor wedge. While entry and exit has been found to amplify aggregate fluctuations (Clementi and Palazzo 2016) and growth (Asturias et al (2016)), its effect on the labor wedge has not been studied. In this section, I show that entry and exit is very important for explaining the significant increase in the labor wedge during the Great Recession. To demonstrate this point, an alternative version of the model without entry and exit to production is considered. In this simpler model, the measure of entrepreneurs is 12% as in the benchmark stationary equilibrium but individuals could not change occupation. To compare with previous results, the tightness of the credit constraint λ_t is selected to generate endogenous decline in aggregate TFP that mimics the TFP path in the benchmark credit crunch experiment. The effects of a credit crunch for this simpler model without entrepreneurial entry and exit is shown in Figure 8 (solid blue curves).

Despite the similar TFP dynamics of the two experiments, the exercise without entrepreneurial entry and exit shows much smaller contractions in output and investment. Also, hours turn out to increase a bit in this exercise while it decreases significantly in the year of the shock in the benchmark experiment.

Figure 10 shows the anatomy of the labor wedge. We can find that the increases in labor wedge is smaller without entry and exit. The movements of the labor wedge is again mostly attributable to the deviation of the real wage from the MRS. Similar to the benchmark economy, the entrepreneurial term d_1 plays a predominant role while the workers' component d_2 has negligible movements. Nevertheless, the increase in entrepreneurs' term d_1 is generated through a different mechanism. In the economy without entrepreneurial entry and exit, d_1 increases upon the arrival of the shock because its denominator $w(1-L)$ have sharper declines relative to its numerator η_1. The analysis of $w(1-L)$ further shows that the increase in d_1 is mainly driven by the reduction in the MPL (due to the tighter collateral constraints) as labor only has modest movements (Fig. 9 bottom left panel).

Comparing to the benchmark credit crunch experiment, the economy without entry and exit no longer have significant movements in d_1 for three reasons. First, entrepreneurs' consumption C_e declines in the year of the shock and then it experiences weak recovery because constrained entrepreneurs cannot exit production. Second, the opportunity cost of leisure, $w(1-\bar{n})\gamma_e$, declines upon the arrival of the shock and then remains stable in year 2 and 3 as entrepreneurs' population, γ_e, is held constant. These two facts imply a small gap

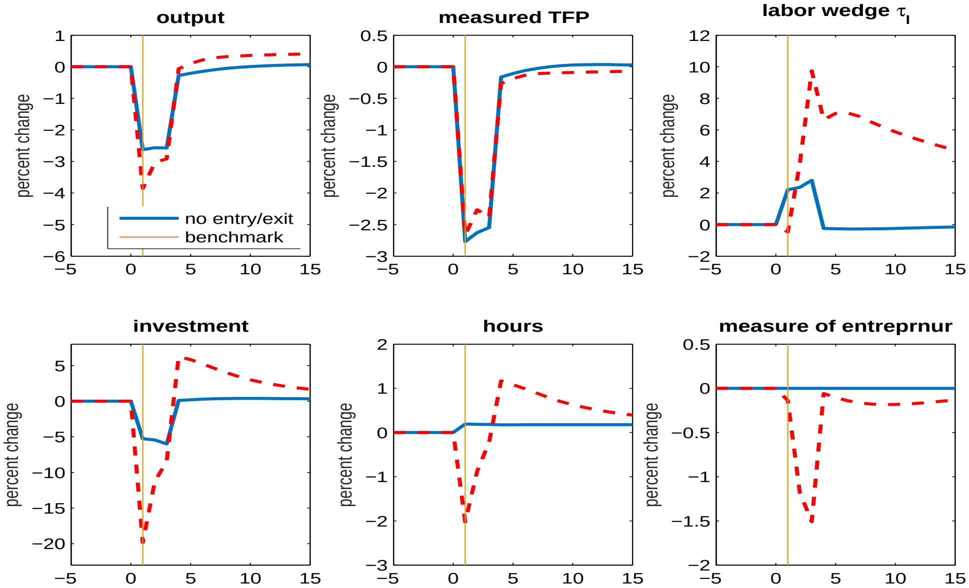

Figure 8: the Role of Entry and Exit in the Credit Crunch

Note.– The shock arrives unexpectedly in year 1 which corresponds to the year 2008. All variable are percent change from steady state.

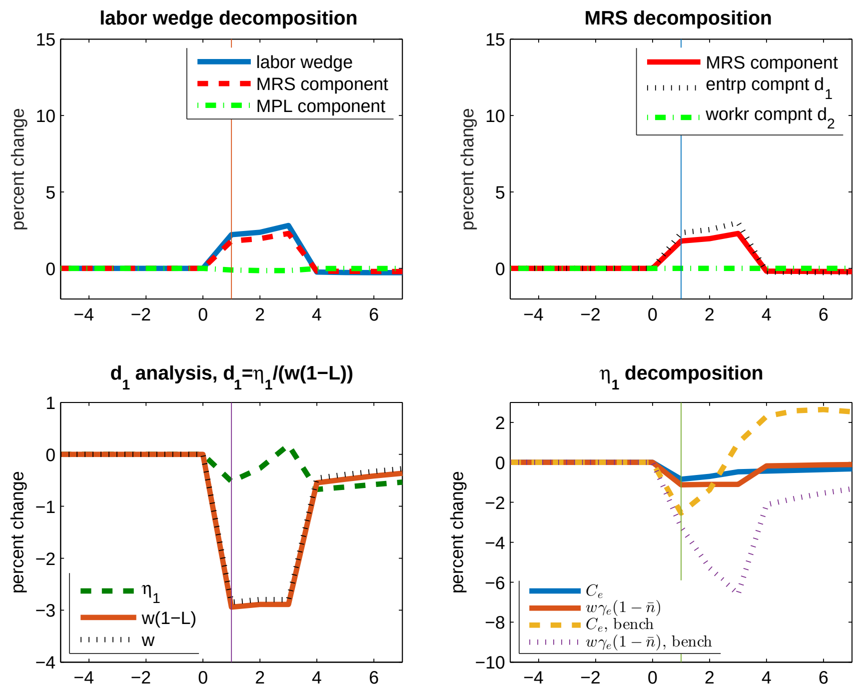

Figure 9: The Labor Wedge Anatomy in the Credit Crunch without Entry and Exit

Note.– The shock arrives unexpectedly in year 1 which corresponds to the year 2008.

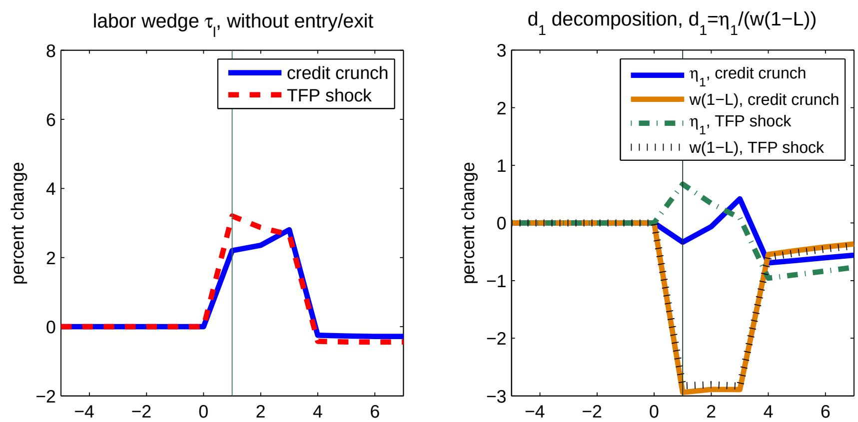

Figure 10: The Labor Wedge in the TFP shock without Entry and Exit

between the real wage and the MRS_e. Third, the denominator, $w(1-L)$, remains stable in year 2 and 3 of the shock even if its initial decline is larger than the benchmark case. These three effects together explain the smaller increases in d_1 (and hence the labor wedge) in the simpler model without entry and exit.

In addition, I conducted an exogenous TFP shock experiment without entry and exit. The z_t series is constructed to mimic the measured TFP path in the credit crunch (Fig. 8, upper middle panel). As shown in Figure 10, the variation of labor wedge becomes very close to that in the credit crunch without entry and exit. In fact, the mechanism of the labor wedge variation becomes very similar in the exogenous TFP shock and the credit crunch: the increases in entrepreneurial deviation d_1 generate increases in the labor wedge and the increases in d_1 is mainly caused by the reduction of the real wage.[26]

In sum, entry and exit of production plays a critical role in the result that the increases of labor wedge in the credit crunch follow a different mechanism from that of an exogenous TFP shock. Without entry and exit, the labor wedge would become much smaller in a credit crunch and the mechanisms for the variation of labor wedge in the credit crunch and the TFP shock become similar.

6 Concluding Remarks

This paper proposes a novel quantitative general equilibrium model to study the impact of a credit crunch on the variation of labor wedge. Individuals have heterogeneous entrepreneurial ability and face discrete occupational choices that allow them to endogenously enter and exit

[26]The only difference between the mechanisms of labor wedge variation in the two shocks is the cause of the real wage decline. In the TFP shock, the real wage falls as aggregate TFP is smaller. In the credit crunch, the real wage decreases because capital input is limited by the tighter collateral constraints. Notice that hours L has little movements in both experiments.

the production sector in each period. Entrepreneurs can produce using capital and labor, but they are subject to collateral borrowing constraints. Also, entrepreneurs work a fixed amount of time while workers can select their labor supply. Under this framework, the labor wedge arises without labor market distortions due to the imperfect aggregation of heterogeneous entrepreneurs and workers. In the paper, the movements of the labor wedge is predominantly driven by the gap between the MRS and the real wage. This is consistent with the empirical findings in Karabarbounis (2014).

I find that a credit crunch can generate severe contractions in economic activity, losses of aggregate productivity and significant increases in the labor wedge. Moreover, when there is endogenous entry and exit of production, a credit crunch and an TFP shock affect the labor wedge through different mechanisms. Consequently, the labor wedge turns out to have larger increases in a credit crunch than that in an exogenous TFP shock. With entry and exit, the tighter collateral constraints in the credit crunch affect entrepreneurs' consumption and opportunity cost of leisure in different patterns, but the exogenous TFP shock does not have this channel. In the credit crunch, the tighter collateral constraints reduce entrepreneurs' profits and consumption. Nevertheless, the exits of low consumption constrained entrepreneurs and the presence of high consumption unconstrained entrepreneurs allow entrepreneurs' consumption to recover rapidly since year 2 of the shock. On the other hand, the tighter collateral constraints limit capital input and keeps the MPL low. With declining entrepreneurial population and real wage, entrepreneurs' opportunity cost of leisure keeps falling. The opposite movements of entrepreneurs' consumption and opportunity cost of leisure together imply a larger deviation of the real wage from the MRS_e. Thus the labor wedge has higher increases in the credit crunch.

The paper also contributes to study the effects of endogenous entry and exit of production on the labor wedge. When entrepreneurial entry and exit is shut down, the labor wedge would have much smaller increases in a credit crunch and the mechanisms for the variation of labor wedge in the credit crunch and the TFP shock are similar. Without entrepreneurial entry and exit, entrepreneurs' consumption only recovers slowly while the opportunity cost of leisure remains stable after the year of the shock. As a result, the gap between the real wage and the MRS_e (and hence the labor wedge) remains small throughout the credit crunch.

This paper has explained about 65% of the increases in labor wedge over the Great Recession using individual heterogeneity and endogenous entry and exit of production. The remaining one-third could be caused by labor market distortions such as searching and matching frictions. Considering the complexity of my current analysis, I leave these to future research.

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Appendices

A Computation

I approximate the continuous *log AR(1)* process for individual ability with a 3 state Markov chain using the procedure in Tauchen (1988). The three ability realizations are $\{0.1353, 0.4500, 1.4965\}$ and the transition matrix is

$$P_\theta = \begin{pmatrix} 0.9010 & 0.0990 & 0.0000 \\ 0.0159 & 0.9681 & 0.0159 \\ 0.0000 & 0.0990 & 0.9010 \end{pmatrix}$$

I solve the steady state of this economy based on the method described in Buera and Shin (2013). The steady state is solved by finding the market clearing real interest rate and real wage. Specifically, the steady state is solved in the following steps:

(1) First guess the l-th generation of real interest rate r^l.

(2) Given r^l, guess wage rate $w^{l,s}$ and solve the individual's problem taken r^l and $w^{l,s}$ as given. After obtaining occupational decisions, asset decision rules and value functions, I use them to find the fixed point of household distribution. Then I check whether the labor market clears. If the labor market do not clear, update wage to a new guess $w^{l,s+1}$ and come back to step (2).

(3) If the labor market clears, I check whether the credit market condition has been satisfied. If not, update the real interest rate to a new guess r^{l+1} and come back to step (2).

(4) The steady state is solved when all markets clear.

The expected value function and households distribution in steady state are inputs for the transitional dynamics.

The perfect foresight transitional dynamics is solved through the following three steps:

(1) Start with steady state value function and solve the value function backwardly from $t = T - 1$ to $t = 1$

(2) Start with steady state distribution, update the household distribution forwardly from $t = 1$ to $t = T$

(3) Check the market clearing conditions in each period to update interest rate and rental price.

The length of time $T = 160$ for the negative productivity shock and $T = 140$ for the credit crunch. The precision for value function $= 1.0e - 4$. The precision for household distribution $= 1.0e - 7$. The precision for market clearing conditions in the steady state and the transitional dynamics $= 1.0e - 3$. All other precision $= 1.0e - 8$.

B Data

Output is the Real Gross Domestic Product (billions of chained 2009 dollars) from Table 1.1.6 of the Bureau of Economic Analysis (BEA). Consumption is Personal Consumption Expenditures less durable goods from Table 1.1.5 of the BEA. Business investment is the sum of durable goods and private nonresidential fixed investment from Table 1.1.5. The real values of consumption and investment are calculated using the GDP deflator. Real GDP, consumption, and business investment are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1969Q1 to 2013Q1. The Solow Residual is measured using data on private capital and hours. Private capital is the sum of private fixed assets and consumer durable from Fixed Asset Table 1.1 of the BEA. Data on hours is constructed by Cociuba, Prescott and Ueberfeldt (2012) and extended to 2014Q4 by the author following their method.

C Robustness

For the robustness of the key results, I also calibrate entrepreneurs in the model as families headed by individuals who claim themselves as self-employed in the 2001 & 2004 Survey of Consumer Finances (SCF). The new calibration results are summarized in Table A.1. Given this new counterpart for entrepreneurs, the moments that determine the three parameters μ_e, ρ_e, σ_{ϵ_e} become: the percentage of entrepreneurs exiting production, the percentage of workers entering production, the Gini coefficient for wealth.

Although the increases of the labor wedge in the benchmark credit crunch experiment become about 8%, the main results of the paper are still valid under this calibration. First, a credit crunch can generate severe contractions in economic activity and losses of productivity. Second, when the economy has endogenous entry and exit of production, the credit crunch affects the labor wedge in a different mechanism from that of an exogenous TFP shock. As a result, the increases of the labor wedge are still higher in the credit crunch than that in the TFP shock. With entry and exit, the tighter collateral constraints in the credit crunch affect entrepreneurs' consumption and opportunity cost of leisure in different patterns, but the exogenous TFP shock does not have this channel. Finally, endogenous entry and exit of production plays a key role in the variation of labor wedge. When entry and exit is shut down, the labor wedge would have smaller increases in the credit crunch and the mechanism for the variation of the labor wedge in the credit crunch is similar to that of an exogenous TFP shock.

D Equilibrium

In equilibrium, the good market clears so that aggregate output Y is equal to the sum of aggregate consumption C, aggregate investment I and aggregate fixed cost payment Ω, i.e. $Y = C + I + \Omega$, where

$$\Omega = \int_{a,\theta} \int_0^{\xi^T(a,0,\theta,S)} \xi G(d\xi) d\mu + \int_{a,\theta} \int_0^{\xi^T(a,1,\theta,S)} \xi G(d\xi) d\mu \tag{34}$$

is the total fixed cost paid by individuals that switch occupations in a period, and

$$Y = \int_{a,\theta} \int_0^{\xi^T(a,0,\theta,S)} f(\theta, z, k, l) G(d\xi) \mu(d[a \times \theta]) + \int_{a,\theta} \int_{\xi^T(a,1,\theta,S)}^{\xi_U} f(\theta, z, k, l) G(d\xi) \mu(d[a \times \theta]) \tag{35}$$

$$\begin{aligned} C = & \int_{a,\theta} \int_0^{\xi^T(a,0,\theta,S)} c^e(a, 0, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{a,\theta} \int_{\xi^T(a,1,\theta,S)}^{\xi_U} c^e(a, 1, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{a,\theta} \int_0^{\xi^T(a,1,\theta,S)} c^w(a, 1, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{a,\theta} \int_{\xi^T(a,0,\theta,S)}^{\xi_U} c^w(a, 0, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \end{aligned} \tag{36}$$

$$\begin{aligned} I = K' - (1 - \delta) K = & \int_{a,\theta} \int_0^{\xi^T(a,0,\theta,S)} a^e(a, 0, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{a,\theta} \int_{\xi^T(a,1,\theta,S)}^{\xi_U} a^e(a, 1, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{a,\theta} \int_0^{\xi^T(a,1,\theta,S)} a^w(a, 1, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) \\ & + \int_{\xi^T(a,0,\theta,S)}^{\xi_U} a^w(a, 0, \theta, \xi, S) G(d\xi) \mu(d[a \times \theta]) - (1 - \delta) \int_{a,\theta,\chi_{-1}} a \mu(d[a \times \theta \times \chi_{-1}]) \end{aligned} \tag{37}$$

The labor market clears so that the total demand of labor from entrepreneurs is equal to the total labor supply from workers.

$$\int_{a,\theta}\int_0^{\xi^T(a,0,\theta,S)} l(a,\theta,S)G(d\xi)\mu(d[a\times\theta]) + \int_{a,\theta}\int_{\xi^T(a,1,\theta,S)}^{\xi_U} l(a,\theta,S)G(d\xi)\mu(d[a\times\theta])$$
$$= \int_{a,\theta}\int_0^{\xi^T(a,1,\theta,S)} n^w(a,1,\theta,\xi,S)G(d\xi)\mu(d[a\times\theta]) + \int_{\xi^T(a,0,\theta,S)}^{\xi_U} n^w(a,0,\theta,\xi,S)G(d\xi)\mu(d[a\times\theta]) \quad (38)$$

The capital market clears so that the aggregate demand of capital equals to the total deposited assets in the economy.

$$\int_{a,\theta}\int_0^{\xi^T(a,0,\theta,S)} k(a,\theta,S)G(d\xi)\mu(d[a\times\theta])$$
$$+\int_{a,\theta}\int_{\xi^T(a,1,\theta,S)}^{\xi_U} k(a,\theta,S)G(d\xi)\mu(d[a\times\theta]) = \int_{a,\theta,\chi_{-1}} a\mu(d[a\times\theta\times\chi_{-1}]) \quad (39)$$

E Deriving the marginal rate of substitution

By the definition of the subsets given in Section 2.7, we have $\mathbf{E}_1\cap\mathbf{E}_2=\emptyset$ and $\mathbf{E}_1\cup\mathbf{E}_2=\mathbf{E}$. Also we have $\mathbf{H}_1\cap\mathbf{H}_2=\emptyset$ and $\mathbf{H}_1\cup\mathbf{H}_2=\mathbf{H}$. From each entrepreneur's and worker's first order conditions, I can have

$$MRS \equiv -\frac{u_2(C,1-L)}{u_1(C,1-L)} \quad (40)$$
$$= \frac{1-\tau}{\tau}\frac{C}{1-L} \quad (41)$$
$$= \frac{1-\tau}{\tau}\frac{\int_{i\in E} c_i di + \int_{i\in H_1} c_i di + \int_{i\in H_2} c_i di}{\int_{i\in E}(1-\bar{n})di + \int_{i\in H_1}(1-n_i)di + \int_{i\in H_2}(1-n_i)di} \quad (42)$$

$$= \frac{1-\tau}{\tau}\frac{\int_{i\in E} c_i di + \frac{\tau w}{1-\tau}\int_{i\in H_1}(1-n_i)di + \int_{i\in H_2} c_i di}{\int_{i\in E}(1-\bar{n})di + \int_{i\in H_1}(1-n_i)di + \int_{i\in H_2}(1-n_i)di} \quad (43)$$
$$= w + \frac{\int_{i\in E}\left[\frac{1-\tau}{\tau}c_i - w(1-\bar{n})\right]di + \int_{i\in H_2}\left[\frac{1-\tau}{\tau}c_i - w(1-n_i)\right]di}{\int_{i\in E}(1-\bar{n})di + \int_{i\in H_1}(1-n_i)di + \int_{i\in H_2}(1-n_i)di)} \quad (44)$$
$$= w + \frac{\eta_1+\eta_2}{1-L} \quad (45)$$

where

$$\eta_1 \equiv \int_{i\in\mathbf{E}}\left[\frac{1-\tau}{\tau}c_i - w(1-\bar{n})\right]di = \frac{1-\tau}{\tau}C_e - w(1-\bar{n})\gamma_e \quad (46)$$

$$\eta_2 \equiv \int_{i\in\mathbf{H}_2}\left[\frac{1-\tau}{\tau}c_i - w(1-n_i)\right]di = \frac{1-\tau}{\tau}C_u - w\gamma_u \tag{47}$$

Equation (60) holds because $\forall i \in \mathbf{H}_1$ we have $\frac{(1-\tau)c_i}{\tau(1-n_i)} = w$.

F Proof of proposition 2

Since $\forall i \in \mathbf{H}_2$ we have $n_i = 0$ and $\frac{(1-\tau)c_i}{\tau(1-n_i)} > w$, thus $\eta_2 > 0$. The sign of η_1 in equation (25) is hard to determine because it depends on the entrepreneurial consumption, the real wage and fixed working time $\bar{n}$. Since $w(1-L) > 0$, we have $d_1 + d_2 \geq 0$ if and only if $\eta_1 + \eta_2 \geq 0$

If in the economy, the measure of entrepreneurs that hire no labor from the market is strictly positive, i.e. $\gamma(E_2) > 0$, then $MPL < w$ according to equation (20). If $\eta_1 + \eta_2 \geq 0$, we have $MRS \geq w$ by equation (25). So the labor wedge $\tau_l = 1 - \frac{MRS}{MPL} < 0$. If $\eta_1 + \eta_2 < 0$, we have $MRS < w$. In this case, the sign of τ_l is indeterminate .

G Production profit function

Solving the entrepreneur's problem, we can obtain the following first order conditions:

$$[k] \quad \alpha\nu z\theta k^{\alpha\nu-1}(\bar{n}+l)^{(1-\alpha)\nu} = (r+\delta) + \frac{\phi_2}{\phi_1}$$

$$[l] \quad (1-\alpha)\nu z\theta k^{\alpha\nu}(\bar{n}+l)^{(1-\alpha)\nu-1} = w - \frac{\phi_3}{\phi_1}$$

where ϕ_1 is the Lagrange multiplier to budget constraint equation (5), ϕ_2 is the Lagrange multiplier to the borrowing constraint equation (6) and ϕ_3 is the Lagrange multiplier to condition $l \geq 0$.

If it is optimal for the entrepreneur to hire labor from the market so that $l > 0$ and $\phi_3 = 0$ then we have profit function π to be

$$\pi(a,\theta,\lambda,z) = (z\theta)^{\frac{1}{1-(1-\alpha)\nu}}\left(\frac{(1-\alpha)\nu}{w}\right)^{\frac{(1-\alpha)\nu}{1-(1-\alpha)\nu}} k^{\frac{\alpha\nu}{1-(1-\alpha)\nu}}\left[1-(1-\alpha)\nu\right] - (r+\delta)k + w\bar{n}$$

where

$$k = \begin{cases} \lambda a & \text{if } \phi_2 > 0 \text{ and collateral constraint binds} \\ \left[\alpha\nu z\theta(r+\delta)^{(1-\alpha)\nu-1}\left(\frac{1-\alpha}{\alpha w}\right)^{(1-\alpha)\nu}\right]^{\frac{1}{1-\nu}} & \text{if } \phi_2 = 0 \text{ and collateral constraint is slack} \end{cases}$$

If it is optimal for the entrepreneur to hire no labor from the market so that $l = 0$ and $\phi_3 > 0$ then profit function becomes

$$\pi(a, \theta, \lambda, z) = z\theta k^{\alpha\nu} \bar{n}^{(1-\alpha)\nu} - (r + \delta)k \tag{48}$$

where

$$k = \begin{cases} \lambda a & \text{if } \phi_2 > 0 \text{ and collateral constraint binds} \\ \left[\left(\frac{\alpha\nu z\theta}{r+\delta}\right) \bar{n}^{(1-\alpha)\nu}\right]^{\frac{1}{1-\alpha\nu}} & \text{if } \phi_2 = 0 \text{ and collateral constraint is slack} \end{cases}$$

Therefore, profit π is a function of (a, θ, λ, z) if $\phi_2 > 0$ so that the collateral constraint is binding. Profit π is a function of (θ, z) when $\phi_2 = 0$ so that the collateral constraint is slack.

After some simple algebra, we can derive that $\frac{\partial \pi(a,\theta,z,\lambda)}{\partial \lambda} > 0$ so that profit function is increasing in λ. With the form of profit function given above, it is also easy to know that $\frac{\partial \pi}{\partial r} < 0$ and $\frac{\partial \pi}{\partial w} < 0$ so that profit is decreasing in input prices.

Suppose $\phi_2 > 0$ so that the collateral constraint is binding and $k = \lambda a$. Then profit is concave in asset a because

$$\frac{\partial^2 \pi(a, \theta, z, \lambda)}{\partial a^2} = \begin{cases} (\alpha\nu - 1)\alpha\nu z\theta\lambda^{\alpha\nu} a^{\alpha\nu-2} (\bar{n})^{(1-\alpha)\nu} & \text{if } l = 0 \\ \frac{(\nu-1)\alpha\nu}{(1-(1-\alpha)\nu)^2} (z\theta)^{\frac{1}{1-(1-\alpha)\nu}} \left(\frac{(1-\alpha)\nu}{w}\right)^{\frac{(1-\alpha)\nu}{1-(1-\alpha)\nu}} \lambda^{\frac{\alpha\nu}{1-(1-\alpha)\nu}} a^{\frac{\alpha\nu}{1-(1-\alpha)\nu}-2} [1 - (1-\alpha)\nu] & \text{if } l > 0 \end{cases}$$

Since α, $\nu \in (0, 1)$, we can know that $\frac{\partial^2 \pi(a,\theta,z,\lambda)}{\partial a^2} < 0$ in both cases. Similarly, we can show that the profit function is concave in λ.

H Tables and Figures

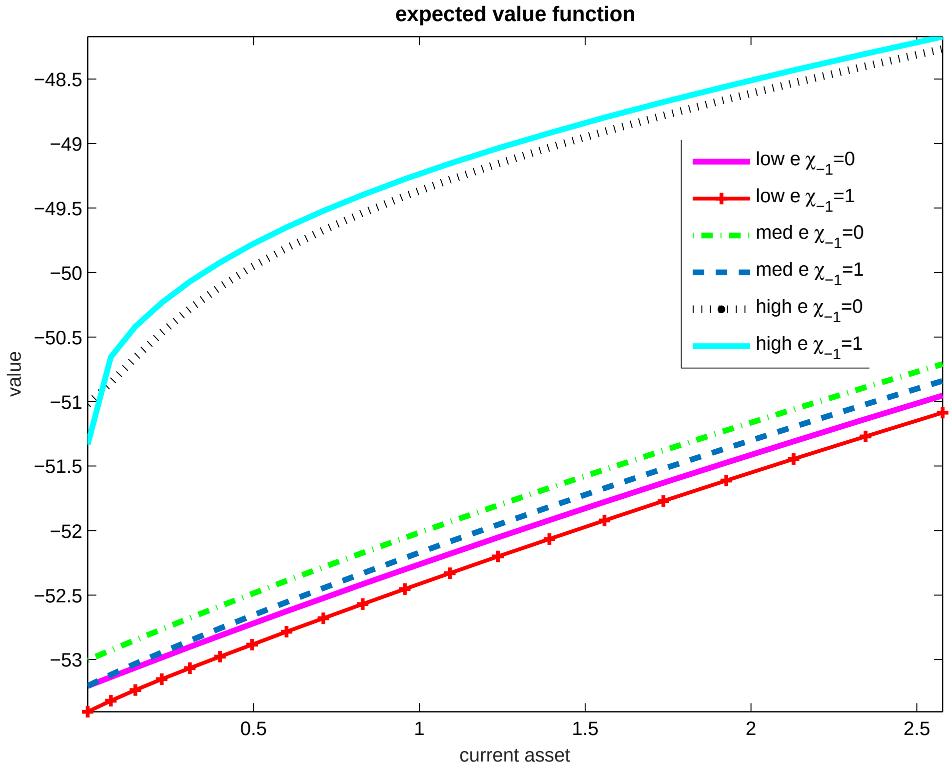

Figure A.1: Expected discounted lifetime value $v^0(a, \chi_{-1}, \theta)$

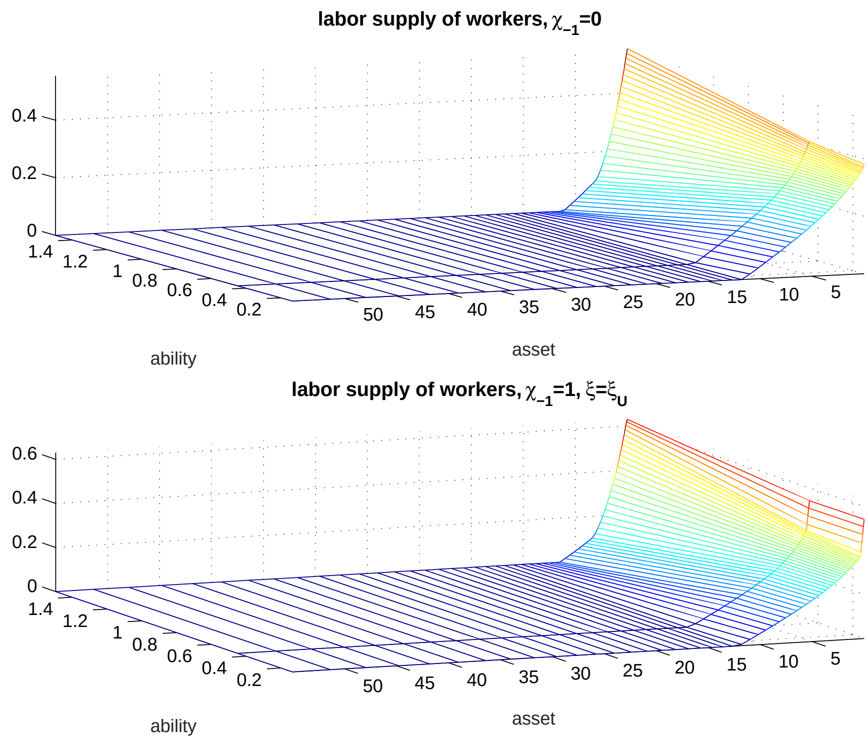

Figure A.2: Labor supply decision

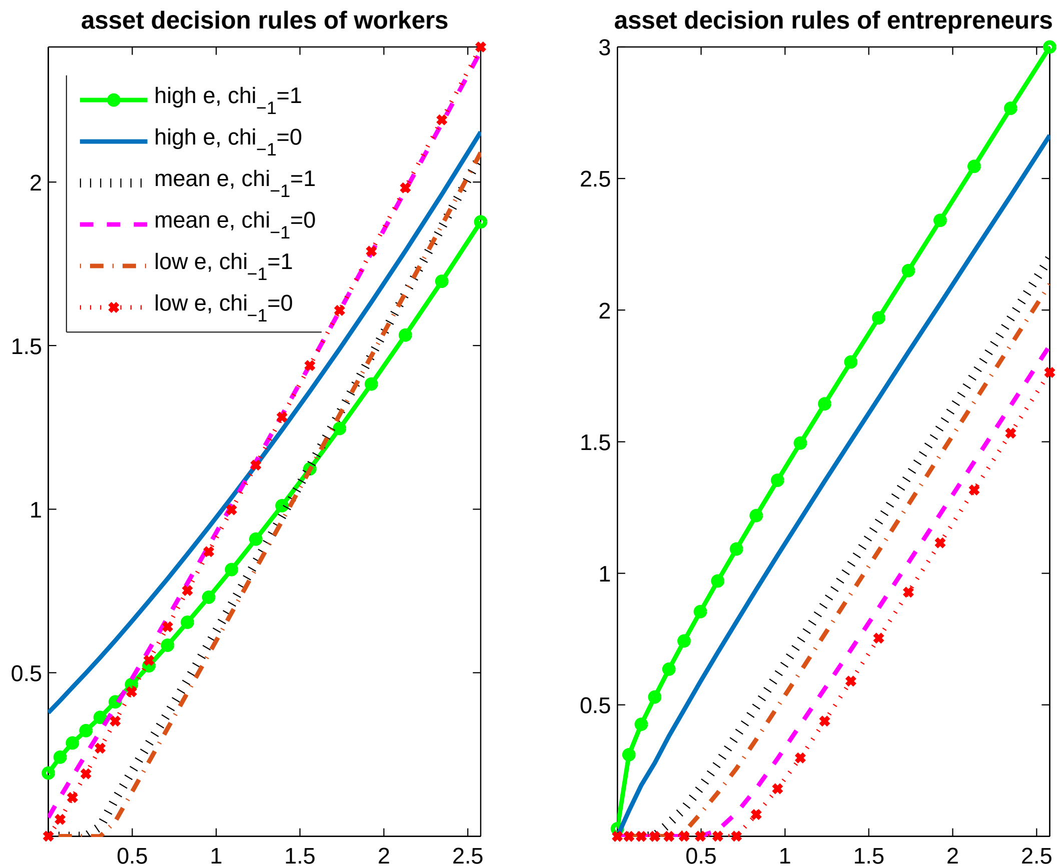

Figure A.3: Asset decision rules of a worker (left) and an entrepreneur (right). For the worker with $\chi_{-1} = 1$ and the entrepreneur with $\chi_{-1} = 0$, the figure shows the asset decisions when they draw fixed cost $\xi = \xi_U$. The asset decision rules for entrepreneurs are derived by solving the entrepreneur's problem in equations (4)-(7) and the asset decision rules for workers are derived by solving the worker's problem in equations (8)-(10).

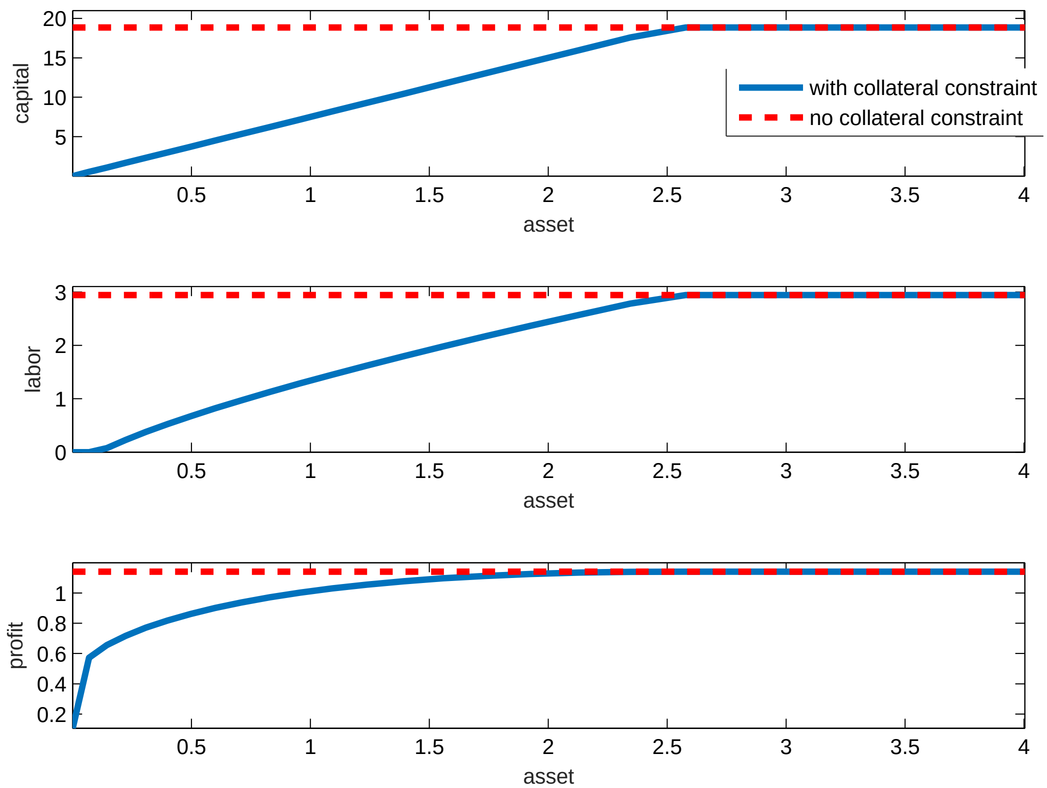

Figure A.4: Steady state production activities

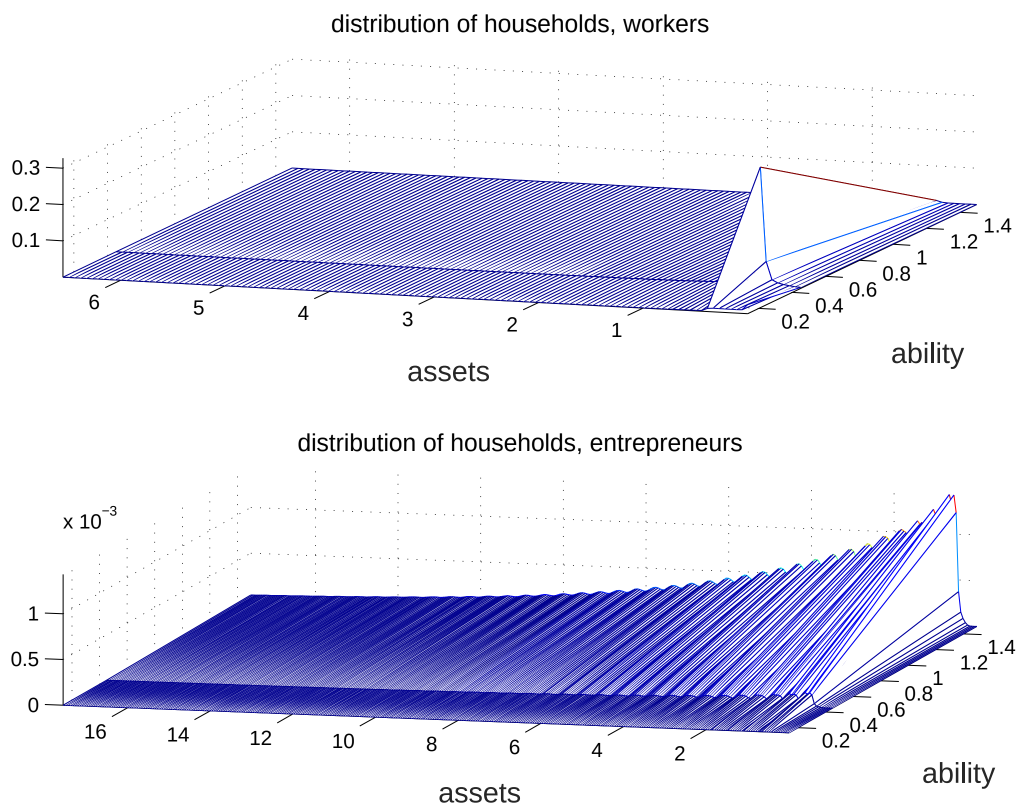

Figure A.5: Households distribution in the steady state

	Target moment	Model	Target	Source
β	Real interest rate	4.0	4.0	U.S. data
τ	Average hours worked	0.33	0.33	U.S. data
α	Capital-output ratio	2.4	2.4	U.S. data
ν	Top 5 percent earnings	31.27	30.0	BS (2013)
λ	External finance to GDP	1.75	1.75	BS (2013)
ξ_U	Percentage of entrepreneurs	11.7	11.7	U.S. data
ρ_e	Percentage of exiting entrepreneurs	22.08	22 to 24	BCD (2015)
σ_e	Percentage of workers entering entrepreneurship	2.92	2.3 to 3	BCD (2015)
μ_e	Gini coefficient for wealth	0.64	0.78	Diaz-Gimenez et al (1997)

Table A.1: Model and Data Generated Moments using Self-employed Individuals as the Counterpart for Entrepreneurs. See section C in the appendix for details.

Note.– Data source: NIPA tables and the Survey of Consumer Finance. BS (2013) refers to Buera and Shin (2013) and BCD (2015) refers to Bassetto, Cagetti and De Nardi (2015)