Heterogeneous Households, Mortgage Debt and Housing Demand over the Great Recession*

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Abstract

This paper studies the contractions in the U.S. housing market and the real economy in the Great Recession. I build a quantitative general equilibrium model with heterogeneous households and two sectors. Households face portfolio problems that involve selecting the stock of housing, mortgage debts and financial assets. The real house price is endogenous and households have the option to default on mortgage debt. The model matches the housing and non-housing moments in the U.S. data. The combination of a negative productivity shock, a financial shock and a housing depreciation shock on the economy can reasonably explain the recessions in the housing market and the real economy. Specifically, the decrease of aggregate productivity accounts for the declines in aggregate output, consumption, investment, labor hours as well as part of the drop in housing production and demand. The tighter financial condition explains the large decreases in mortgage debt, leverage and housing demand. The high housing depreciation generates a high mortgage default rate and reduces housing demand as household net worth shrinks. Households deleverage and reduce mortgage debt when the financial condition is tightened or when housing value is exposed to persistently high depreciation shocks because the cost of borrowing through mortgages would increase greatly relative to the benefit of borrowing in either case.

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1 Introduction

Understanding the movements of macroeconomic housing variables is important for explaining the Great Recession as it started with the housing market. From 2002 to 2006, the U.S. housing market experienced a rapid increase in housing production and house prices, which contributed to the economic boom from 2003-2007. The housing market collapsed in 2006 and the high foreclosure rate in the residential mortgage market was followed by the so called Great Recession. Figures 1 and 2 contain the time series on key housing market and macroeconomic variables. First of all, the housing market experienced a long depression. From 2006Q3-2011Q4, real house prices and real housing wealth declined about 25%, residential investment decreased 45% and real aggregate mortgage debt fell about 18%. Household leverage in the housing market dropped 17% between 2009Q1 and 2013Q1.[1]Secondly, the housing downturn was accompanied by a severe contraction in real economic activity outside the housing market. Specifically, real output was about 5% lower than its 2007Q4 level in early 2009. Hours dropped 10% and consumption decreased 4.5% from 2008Q2 to 2009Q2. Finally, the residential mortgage foreclosure rate increases significantly after 2006Q2.

How can we understand the recessions in the housing market and the real economy? Can the patterns noted above be explained by a productivity shock? How do the financial tightening and the higher foreclosure rate in the residential mortgage market affect the housing and non-housing variables? These questions have not been answered yet as existing literature either aims to explain only the real economic recession or study some specific targets in the housing market.[2] The objective of this paper is to reproduce the housing recession alongside the real economic recession, and explore the effects of financial tightening and housing foreclosure boom on the aggregate economy in a dynamic stochastic general equilibrium (DSGE) model with heterogeneous households and mortgage default.

Specifically, the model is populated by infinitely lived heterogeneous households with idiosyncratic labor productivity that solve consumption, labor and portfolio problems each period. The portfolio problem involves selecting the stock of housing, mortgage debt and non-housing/financial assets to maximize expected lifetime value of consumption, housing service and leisure. Houses can serve as collateral to borrow through mortgages. Nevertheless, households have the option to default on mortgage debts. Financial intermediaries issue mortgages and price them in the way such that household default risk and the fluctuations of real house prices are fully reflected. To allow for variations in real house prices, my model includes two sectors: a consumption good sector and a housing good sector. The model is

[1]Leverage in this paper is defined as the ratio of real mortgage debt to real housing value.

[2]Real economic recession means the declines of real output, consumption, labor hours and investment in this paper.

calibrated to match the empirical housing and non-housing moments. With the parameters identified in calibration, the benchmark model is able to generate homeownership rate, non-housing asset to housing asset ratio, housing wealth to GDP ratio and housing investment to GDP ratio that are close to their correspondences in the U.S. data.

To explain the patterns of the data shown in Figures 1 and 2, I have done several experiments. A *persistent* negative productivity shock in both housing and consumption good sectors can generate a real economic recession with contractions of housing output and demand. However, the pure negative productivity shock fails to explain the decrease of mortgage debt and leverage in the data. To fix these housing market variables, I keep the same productivity shock and introduce a *permanent* tightening in housing finance and a *one-period* housing depreciation shock to mimic the environment in the Great Recession. With these three shocks to the economy, the real economic recession is maintained and the movements of housing variables in the data can be explained. Specifically, the decrease of aggregate productivity in the two sectors accounts for the declines of aggregate output, consumption, business investment, labor hours as well as part of the drop in housing production and demand. The tighter housing finance condition is responsible for the large decreases in mortgage debt, leverage and the decline of housing demand. The *one-period* high housing depreciation generates high mortgage default rate and reduces housing demand as household net worth shrinks. Thus this Foreclosure Boom Experiment with a *persistent* negative productivity shock, a *permanent* housing financial tightening and *one-period* housing depreciation shock can reasonably explain all of the housing and non-housing variables understudy except the dynamics of the real house prices.[3][4]

The impact of the high foreclosure rate in the residential mortgage market on the aggregate economy is explored in the experiment with a *persistent* housing depreciation shock. The shock generates a persistent housing foreclosure boom and also leads to the decreases in mortgage debt, leverage, housing demand and production. However, the *persistent* housing depreciation shock is not able to account for the real economic recession.

The model has four key ingredients. Firstly, when households default the only punishment is to lose the ownership of the houses. There is no recourse state and no transaction cost in housing purchase and selling. Also, households are not discriminated in the financial markets if they default. As a result, the default option is chosen if and only if the housing asset is underwater, i.e. the realized housing value is smaller than the mortgage loan value. This "ruthless default" rule simplifies the mortgage price schedule and reduces the number

[3]The objective of this paper is to explain the contraction in both the real economy and the housing market as much as possible. Understanding the dynamics of real house prices in the data is not the emphasis and goal of this paper.

[4]The experiment with negative productivity shock and tighter finance condition can explain the housing and non-housing variables as good as the three shock experiment except the movements of foreclosure rate.

of individual states that are needed to be tracked so that the computational burden of solving the model is greatly reduced. Secondly, houses can be rented out for rental incomes but they are risky assets as they are exposed to idiosyncratic depreciation shocks.[5] Since the house is seized by the banks when household default and there is no recourse state, households also save using risk-free financial asset to smooth consumption even if it has a lower return.[6] Moreover, the uncertainty in housing return because of the idiosyncratic depreciations generates mortgage default with the stable real house prices in the steady state. Thus a realistic foreclosure rate in the residential mortgage market can be matched by calibrating the invariant distribution of the depreciation shock. Thirdly, issuing mortgage is costly so that the banks lose an additional $r_w > 0$ units of real resource per unit of mortgage debt issued. Parameter r_w measures the productivity of mortgage insurance sector and it also breaks up households' indetermancy between saving and borrowing as the risk-free mortgage interest rate is strictly higher than the real interest rate.[7] A financial tightening in the paper is modeled as an increase in r_w to characterize the lower efficiency in mortgage issuance. Finally, the real house prices, with consumption good as the *numeraire*, is endogenously determined as I model two sectors with two factors.

In the Foreclosure Boom Experiment with a *permanent* tightening in housing finance, a *one-period* depreciation shock and a *persistent* negative productivity shock in both sectors, the decrease of productivity accounts for the declines in aggregate output, consumption, investment and labor hours. Housing demand shrinks for two reasons. First, households who experience large reductions in their labor income demand less housing assets as they reduce net saving to smooth consumption. Second, since the financial tightening makes borrowing through mortgages more costly, households borrow less and reduce housing demand to avoid larger down payment requirements for the same houses.

The unexpected decline in aggregate productivity generates a deep recession in both the consumption good sector and the housing good sector, but the housing recovery is several periods later than that in the consumption good sector. Housing output does not recover right after the shock hits because the factor price ratio becomes such that the capital and labor inputs in the housing sector decrease further after the shock hits. Therefore, the postponed recovery in the housing good sector is a general equilibrium result.

With tighter housing finance condition, households find it optimal to decrease leverage and mortgage debts. Two points are important to understand the substantial decline in mortgage

[5]The depreciation shock is i.i.d. drawn from an invariant distribution. In this paper housing depreciation captures the fluctuations in housing value.

[6]Rental income is the only source of housing return in the steady state as house prices are stable. In the benchmark economy, rental income generates a housing return that is higher than the risk-free interest rate.

[7]See Jeske, Mitman and Krueger (2013) for more details.

debt and the mechanism of the household deleveraging process. First, households save using both risk-free financial assets and risky housing assets, even if financial assets yield lower return. When households are hit by big depreciation shocks or experience large unexpected declines in real house prices so that their housing assets are underwater, they default and their net worth hinges on their holdings of financial assets, which cannot be seized by the banks. Thus the low return risk-free financial assets help households to smooth consumption in case of default. Second, the optimal leverage/mortgage down payment policy is determined by household valuations over the benefit and cost of borrowing. Given housing assets, lower down payment/higher debt allows household to spend less in purchasing houses, save more in financial assets and increase consumption. So the benefit of borrowing through mortgages is the increase in household value via raising and smoothing consumption. The cost of borrowing through residential mortgages is the decrease in household value caused by higher default risk and larger net interest payments to the banks. The optimal leverage ratio is such that the benefit of borrowing equates to its cost. When the financial condition becomes tighter, households find that the cost of borrowing through mortgages has greatly increased relative to its benefit as default risk and net interest payment to the banks increase. Thus, in the financial crisis households avoid the higher cost of borrowing by reducing leverage and their holdings of mortgage debts. Given smaller leverage, households' demand for financial assets also decline.[8]

This paper is first related to business cycle models with home production. Leading examples are Davis and Heathcote (2005), Iacoviello and Neri (2010), and Greenwood and Hercovitz (1991). These papers study multi-sector production and can match housing investment data well. However, they do not distinguish owning and renting, do not model mortgage default and household heterogeneity in housing and non-housing wealth, and thus cannot match the homeownership rate, foreclosure rate and household wealth distribution in the data.

This paper is closely related to papers that study the housing market with heterogeneous agents such as Gervais (2002), Chatterjee and Eygungor (2010), Corbae and Quintin (2012), and Jeske, Krueger and Mitman (2013). Gervais (2002) studies the distortions of housing taxation on the composition of aggregate capital. Jeske et al (2013) builds a heterogeneous agents model with an endogenous mortgage default option to study the macroeconomic and distributional impact of subsidies from Government Sponsored Enterprises. They find that eliminating the subsidies leads to a substantial reduction of mortgage origination and increases aggregate welfare. Their insightful paper provides a useful framework on housing

[8]The U.S. household financial asset declined 15.1% from 2007Q4 to 2008Q4.

and mortgage markets with collateralized default and mortgage pricing. However, Jeske et al (2013) is silent about the macroeconomic contractions in the housing market and the real economy in the recent financial crisis. My paper models the same household portfolio problem and a similar mortgage issuance sector as in Jeske et al (2013), but it differs from Jeske et al (2013) in three key respects. Firstly, the real house prices are endogenous in the model. Secondly, this paper departs from their endowment economy setting to include two sectors that produce consumption goods and housing goods respectively. Thirdly, this paper aims to study the impact of aggregate productivity, financial tightening and the high housing foreclosure rate on the aggregate economy and the housing market in order to understand the Great Recession.

Iacoviello and Pavan (2013) stands between the previous two streams of housing literature. They study the procyclicality of housing demand and mortgage debt over the business cycle and find that higher individual income risk and lower down payments can explain the reduced volatility of housing investment, the reduced procyclicality of debt and part of the reduced volatility of GDP. My model also succeeds in generating declines in housing demand and mortgage debt and is thus complementary to Iacoviello and Pavan (2013). However, this paper extends Iacoviello and Pavan (2013) to model mortgage default and explore the implications of real and financial shocks on a comprehensive range of housing and non-housing variables such as housing production, foreclosure, aggregate output, consumption and investment.

To the best of my knowledge, mine is the first paper that has studied the housing and real economic activities with mortgage default and endogenous house prices in a heterogeneous agent framework. In addition, the rich elements in the model allow me to study the effect of an increase in the residential housing foreclosure rate on the aggregate economy. To explain the Great Recession, it is important to understand the aggregate implications of high foreclosure rate because the high foreclosure in the residential housing market has been widely believed to be related to the cause of the 2007-2009 financial crisis. The existing literatures have not explored this point.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 discusses parameterization. Section 4 summarizes the steady state results. Section 5 considers the transitional dynamics of a pure technology shock. Section 6 presents the results of the financial tightening and negative productivity shock. Section 7 shows the results of the Foreclosure Boom Experiment. Section 8 discusses the dynamics resulting from a persistent housing depreciation shock. Section 9 concludes.

2 The Model

2.1 Demographics and the Default Decision Rule

There is a continuum of households that are indexed by $i \in [0,1]$. Each household is endowed with one unit of time to divide between labor and leisure. Households live infinitely and have idiosyncratic labor productivity ϵ. Households save using two kinds of assets. Firstly, households can hold risk-free non-housing/financial asset a which earns risk-free interest rate r per unit of assets saved. Secondly, households can purchase perfectly divisible housing asset h. However, houses are risky assets as they are subject to idiosyncratic housing depreciation shocks. Let δ' denote the housing depreciation shock tomorrow. Depreciation δ' is an i.i.d. draw across time for every household from the continuously differentiable cumulative distribution function $F(\delta')$, $\delta' \in [\underline{\delta}, 1]$.

There is a competitive housing rental market where households can trade housing services. One unit of housing asset generates one unit of housing service. A house purchased at the beginning of a period can be rented out immediately and thus generate rental income in the same period as the purchase. Short selling of risk-free non-housing assets and houses are prohibited.

Households can use housing assets as collateral to take out mortgages issued by the banks. Let m' denote the size of the mortgage, and p_m denote the mortgage price. A household that enters the next period with (h', m')[9] has the option to default on its mortgage payment after observing the housing price p'. If the household chooses to default, the punishment is losing the ownership of the house to the banks. A defaulting household is not punished in any other form in the financial market. There is no recourse state and no transaction cost in housing purchases and sales. Given these assumptions, a household chooses to default if and only if the housing asset is underwater, i.e., if the housing value is smaller than the mortgage payment. That is,

$$p'(1-\delta')h' < m' \tag{1}$$

Equation (1) is the household default decision rule. It implies that the ex-ante default probability at the origination of the mortgage prior to observing the depreciation only depends on the size of the mortgage m' and housing value $p'h'$.[10] Thus mortgage price p_m is simply a function of (m', h'). It also implies that the cutoff housing depreciation rate at which a household is indifferent between defaulting and repaying is $\delta^* = \max\left\{\underline{\delta}, 1 - \frac{m'}{p'h'}\right\}$.

[9] h', m' are household state variables for the next period. I do not need to keep track of them in the household problem because it is household net worth that determines household consumption and saving decisions. The simple default decision rule here provides a clean relationship among housing, mortgage and net worth: $x' = (1+r')a' + \max\{0, p'(1-\delta')h' - m'\}$

[10] See Jeske et al (2013) for a more detailed discussion.

2.2 Household problem

Let x denote household net worth which is the real value of all assets brought into the period after the housing depreciation shock has materialized. Let ϵ denote the household labor productivity. Households thus have two individual state variables (x, ϵ). Let $\mu(x, \epsilon)$ denote households distribution over individual state variables (x, ϵ). Then aggregate state variables are (z, μ), where z represents aggregate productivity. Since the main interest of the paper is the stationary economy and perfect foresight transitions, the dependence of prices on (z, μ) are left implicit. In each period, households maximize discounted expected lifetime value from consumption c, leisure $(1 - n)$, and housing services s taking real interest rate r, real wage rate w, real rental price p_s and mortgage price function $p_m(\cdot, \cdot)$ as given. That is, households solve the following problem

$$V(x, \epsilon) = \max_{c,s,a',h',m',n} u(c, s, 1 - n) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) \int_{\underline{\delta}}^{1} V(x', \epsilon') dF(\delta') \tag{2}$$

subject to

$$c + p_s s + a' + ph' - m' p_m(m', h') = w\epsilon n + x + p_s h' \tag{3}$$

$$c, s, a', h', m' \geq 0, \quad 0 \leq n < 1 \tag{4}$$

where net worth $x' = (1 + r')a' + \max\{0, p'(1 - \delta')h' - m'\}$

Equation (3) is the household budget constraint. The right hand side of equation (3) denotes resources available to the household within the period including labor income and net worth. Since the timing is that houses purchased this period can be rented out immediately, household rental income $p_s h'$ also shows up as part of the household resources within the period. The left hand side of equation (3) is the household allocation of resources among consumption, housing service and asset portfolio which involves selecting the level of financial assets, housing assets and mortgage debt.

Future net worth x' consists of financial asset income and home equity. If future housing value after the realization of housing depreciation is larger than the mortgage debt, home equity is positive and equals $p'(1 - \delta')h' - m'$. In this case, households repay the debts. Otherwise, household home equity is zero and net worth $x' = (1 + r')a'$ as households use the default option and have their houses foreclosed on.[11]

[11] In reality, it is possible that home equity becomes negative and households do not trigger default for various reasons. For example, there are penalties on household credit report if they default. Homeownership itself might be valuable to the households and it involves losses of additional resources to move to new places. In the model, existing assumptions eliminate these possibilities and default is chosen if and only if the housing asset is underwater. Thus, home equity is always nonnegative in the model.

2.3 The Banking Sector

Assume that banks are perfectly competitive and have the technology to convert risk-free assets into productive capital without any cost. At the beginning of each period, banks take deposits of financial assets from households, lend capital to the housing production sector and issue mortgages. Following Jeske et al (2011), I assume that issuing mortgage is costly so that banks have to lose an additional r_w units of real resources per unit of mortgage issued. r_w characterizes the screening, monitoring, administrative as well as maintenance costs associated with each unit of mortgage. Thus the effective cost of issuing a unit of mortgage equals $r + r_w$ and banks discount the expected payments received next period at $\frac{1}{1+r+r_w}$.[12] When households default, banks seize the after depreciation housing value. However, the bank foreclosure process is costly and only recovers a fraction $\theta \in [0, 1]$ of the collateral value.

Banks take into account that households might default on the mortgage payments next period. Therefore, mortgage price is such that each mortgage contract compensates for the expected loss in the case of default.

$$m'p_m(m', h') = \frac{1}{1 + r + r_w} \left\{ m'F(\delta^*) + \theta p'h' \int_{\delta^*}^{1} (1 - \delta')dF(\delta') \right\} \tag{5}$$

where $\delta^* = \max \left\{ \underline{\delta}, 1 - \frac{m'}{p'h'} \right\}$ is the cutoff housing depreciation rate at which a household is indifferent between defaulting and repaying.

In equation (5), $m'p_m(m', h')$ is the actual units of consumption that a household obtains when he takes out a mortgage of size m' and buys a house of size h'. The right hand side is the expected discounted revenue that banks receive next period from (m', h'). With probability $F(\delta^*)$ household receives a housing depreciation shock δ' that is lower than the threshold depreciation δ^* so that repay the mortgage is optimal. With probability $1-F(\delta^*)$ households default and banks liquidize the house after a costly foreclosure process which only recovers θ fraction of the after depreciation housing value.

2.4 Representative Production Sectors

There are two representative production sectors in the economy, a consumption good sector and a housing good sector. Assume that capital and labor are perfectly mobile and the aggregate productivity z is the same across sectors.

The consumption good sector produces consumption goods using capital and labor ac-

[12] I follow Jeske et al (2013) to assume that r_w is paid when the mortgage is repaid. When a household defaults on a mortgage payment, it also defaults on the mortgage issuance cost.

cording to production technology $Y_c = zK_c^{\alpha}N_c^{1-\alpha}$. Thus, the representative consumption good firm solves the following problem

$$\max_{K_c, N_c \geq 0} \left\{ zK_c^{\alpha}N_c^{(1-\alpha)} - (r + \delta_k)K_c - wN_c \right\} \tag{6}$$

The housing sector produces new homes using capital and labor according to production technology $I_h = zK_h^{\nu}N_h^{1-\nu}$. Let δ_k denote capital depreciation and p be the real housing price with consumption good as the *numeraire*. The representative firm in the housing sector solves

$$\max_{K_h, N_h \geq 0} \left\{ pzK_h^{\nu}N_h^{1-\nu} - (r + \delta_k)K_h - wN_h \right\} \tag{7}$$

The above two static maximization problems imply that profits are maximized by choosing K_c, N_c, K_h N_h such that

$$r = \alpha z K_c^{\alpha-1}N_c^{1-\alpha} - \delta_k \tag{8}$$

$$w = (1-\alpha)zK_c^{\alpha}N_c^{-\alpha} \tag{9}$$

$$p = \frac{1}{\nu z}\left(\frac{\nu}{1-\nu}\right)^{1-\nu}(r + \delta_k)^{\nu} w^{1-\nu} \tag{10}$$

2.5 General Equilibrium

A recursive competitive equilibrium consists of a set of functions

$$(p, p_s, p_m, r, w, V, c, s, n, a', h', m', N_c, N_h, K_c, K_h, \mu) \tag{11}$$

that satisfies the following conditions.

(i) Given prices p, p_s, p_m, r and w, the value function V solves (2) and c, s, n, a', m', h' are the associated policy functions

(ii) Given prices, policies K_c, N_c solves the consumption good production problem and N_h, K_h solves the housing production problem

(iii) Given $p_m(\cdot, \cdot)$, financial intermediaries break even for all (m', h')

(iv) The consumption good market clears

$$\int c d\mu + I_b = Y_c \tag{12}$$

where $I_b = I + (r_w + r_{-1} - r)\int p_m(m, h) m d\mu$ is the broad investment and $I = \bar{K}' - (1 - \delta_k)\bar{K}$ is the investment in the aggregate capital stock. r_{-1} is the real interest rate in the previous period and $\int p_m(m, h) m d\mu$ is the aggregate mortgage loan that is borrowed by households in the previous period.[13] $\bar{K}$ is the aggregate capital stock.

(v) The housing rental market clears

$$\int s d\mu = \int h' d\mu \tag{13}$$

(vi) The labor market clears

$$N_c + N_h = \int (\epsilon n)\, d\mu \tag{14}$$

(vii) The asset market clears

$$\int a' d\mu = \int p_m(m', h') m' d\mu + \bar{K}' \tag{15}$$

(viii) The capital market clears

$$K_c + K_h = \bar{K} \tag{16}$$

(ix) The housing market clears

$$\int h' d\mu = I_h + H \tag{17}$$

where I_h is the total newly built houses this period and $H = \int \int_{\underline{\delta}}^{\delta^*} h(1-\delta) dF(\delta) d\mu + \int \theta \int_{\delta^*}^{1} h(1-\delta) dF(\delta) d\mu$ is the effective aggregate housing stock after depreciation and the foreclosure process.

(x) The evolution of household distribution over individual variables, $\mu(x, \epsilon)$, is consistent.

3 Parameterization

One period in the model is a quarter. The momentary utility function for household is

$$u(c, s, 1-n) = \frac{\left(c^{\tau_1} s^{\tau-\tau_1} (1-n)^{1-\tau}\right)^{1-\sigma} - 1}{1-\sigma} \tag{18}$$

Table 1 lists the parameters that are adopted directly from the data. Suppose the idiosyncratic labor productivity ϵ follows a log AR(1) process

$$log\epsilon_{t+1} = \rho_\epsilon log\epsilon_t + (1-\rho_\epsilon^2)^{0.5} \eta_{\epsilon,t}, \quad \eta_\epsilon \sim N(0, \sigma_\eta^2) \tag{19}$$

[13] The quantity $\int p_m(m, h) m d\mu$ does not depend on $p_m(\cdot, \cdot)$. Actually, this is the quantity that pins down the price $p_m(\cdot, \cdot)$. The notation should not cause confusion.

I follow Jeske et al (2013) to set the persistence of labor productivity $\rho_\epsilon = 0.98$ and the standard deviation $\sigma_\eta = 0.3$, which are in line with the empirical literature on labor productivity and a vast literature on the nature and specification of the household income process. In Pennington-Cross (2004), the estimates of the average default loss is 22% with national data. I let $\theta = 0.78$ to be consistent with Pennington-Cross (2004). I follow Jeske et al (2013) to set the CRRA parameter $\sigma = 3.9$.

To generate a realistic housing foreclosure rate in the steady state, the housing depreciation process $F(\delta)$ is assumed to be a Pareto distribution with probability density function

$$f(\delta) = \frac{1}{\sigma_\delta}\left(1 + \frac{\gamma(\delta - \underline{\delta})}{\sigma_\delta}\right)^{\left(-\frac{1}{\gamma}-1\right)} \tag{20}$$

I calibrate the three parameters γ, $\underline{\delta}$ and σ_δ by targeting three moments in the data: the mortgage foreclosure rate, mean depreciation of residential fixed assets and the standard deviation of housing prices. According to the National Delinquency Survey from Mortgage Banker Association (MBA (2006)), the average quarterly foreclosure rate of all mortgage loans is about 0.4% from 2002Q1 to 2006Q4 . The mean depreciation for residential housing is calculated as the consumption of fixed capital in the housing sector divided by the total capital stock of residential housing. The data on the consumption of fixed capital in the housing sector is taken from Table 7.4.5 of National Income and Product Account (NIPA), and the capital stock of residential housing is taken from Fixed Asset Table 1.1.[14] My estimation of the quarterly mean depreciation for residential housing is 1.4%.[15] The standard deviation of the housing value is obtained by utilizing the state volatility parameter from the Federal Housing Finance Agency (FHFA or OFHEO). The state volatility parameter, which is measured using sales prices only, reflects the standard deviation of housing price growth after four quarters from 1991Q1 to 2013Q2. According to the FHFA, the standard deviation of housing prices in the 50 states and the District of Columbia of the United States varies from 6-9% and has a mean value about 8%. Therefore, I choose the volatility target to be 8%.

I choose parameter $\tau = 0.45$ so that households in the model on average work one-third of their time. $\tau_1 = 0.385$ is chosen so that the share of housing in total consumption expenditure is 14.4%, which is measured using annual data from 1969 to 2001 (NIPA Table 2.4.5). As shown in Figure 1, real mortgage debt is about 0.41 times as large as the housing wealth from 1969Q1 to 2002Q4. The mortgage administration cost $r_w = 0.0001$ is adopted so that the aggregate leverage ratio in the steady state is very close to this target. The time discount

[14] Table 7.4.5 published by BEA June 25, 2010

[15] This estimation stands in line with Macro-Housing literature such as Jeske et al (2011) and Iacoviello and Pavan (2013).

factor $\beta = 0.978$ is chosen to hit a quarterly real interest rate of 1.2% in the steady state.

On the production side, I set parameter $(1-\nu) = 0.88$ to match labor's share in the construction sector. The average labor's share in the construction sector from 1987 to 2002 is measured to be 0.88 using the method and data source provided in Davis and Heathcote (2005).[16] I choose the capital's share in the consumption good sector $\alpha = 0.301$ such that the aggregate capital to aggregate output ratio $\frac{K}{Y}$ is 3.07 in the steady state and the consumption capital to consumption output ratio $\frac{K_c}{Y_c}$ is about 2.0, which are consistent with the U.S. data. I choose the capital depreciation rate $\delta_k = 0.025$ to be consistent with King and Rebelo (2000). I assume that the aggregate productivity is the same in the consumption good sector and the housing sector. Aggregate productivity z follows a log AR(1) process

$$log(z_{t+1}) = \rho_z log(z_t) + \zeta_t, \quad \zeta_t \sim N(0, \sigma_\zeta^2) \tag{21}$$

where $\rho_z = 0.95$ as in Bloom et al (2011) and $\sigma_\zeta = 0.0072$ as in King and Rebelo (2000).

4 The Steady State

In this section, I illustrate the steady state properties of the model. Figure 3 plots the mortgage price function $p_m(m, h)$ provided by the banking sector as described in equation (5). Since $F(\delta)$ is a continuous differentiable distribution, $p_m(m, h)$ is also continuous and differentiable in m and h. As shown in Figure 3, the mortgage price is higher when a larger house h is pledged as collateral, holding mortgage m constant. Given the housing asset h, mortgage price decreases as mortgage debt m increases.

Actually, mortgage price is simply determined by household leverage. Let $\iota = \frac{m}{ph}$ denote the leverage ratio, then equation (5) can be rewritten as[17]

$$p_m(\iota) = \frac{1}{1+r+r_w}\left\{F(1-\iota) + \frac{\theta}{\iota}\int_{1-\iota}^{1}(1-\delta)dF(\delta)\right\} \tag{22}$$

By the definition, selecting the optimal mortgage debt is equivalent to choosing the optimal leverage ratio, holding houses h and housing price p constant. Taking derivative with respect to ι, one can find that $p_m'(\iota) < 0$. Thus the mortgage price is monotonically decreasing in leverage. Intuitively, since the default probability is equal to $1 - F(1-\iota)$, a larger leverage implies a higher probability of default as the threshold depreciation rate, which is equal to

[16] I abstract land as a production factor in the housing sector. I estimate the capital output ratio in the construction sector to be about 0.16. Thus capital here is more appropriate to be interpreted as the combination of capital and land since physical capital itself is almost negligible in the construction sector.

[17] Proposition 2 in Jeske et al (2011) implies that it is never optimal for households to choose leverage $\iota > 1 - \underline{\delta}$ in equilibrium. Thus the threshold depreciation $\delta^* = 1 - \iota$ without loss of generality.

$1 - \iota$, becomes lower.

Figure 4 plots the value function over net worth and labor productivity. A household's value is higher the larger its net worth and/or labor productivity. Let $g' \equiv a' + (p - p_s)h' - m'p_m(h', m')$, then g' is the net saving from households.[18] By solving a consumption-leisure-savings problem illustrated in the appendix, I find that the net saving policy is linear and increasing in net worth and labor productivity, which is shown in Figure 5.

Figure 6 shows the housing decision as a function of net worth and labor productivity. Larger net worth and labor productivity means more resources are available to households to allocate between different assets. Under the model framework, households find it optimal to buy a larger house if they have larger net worth. Although households can obtain housing service from renting, households demand risky housing assets because they yield higher expected return than the financial assets. Specifically, the expected return of housing investment comes from two sources: the implicit rental income and the potential appreciation in home value. Since house prices are constant in the steady state, the unique source of housing return is the rental income.[19]

Figure 7 shows that household leverage policy decreases monotonically with net worth and/or labor productivity. Leverage is high (at close to 50%) for households with little wealth. Leverage then drops quickly as net worth increases until it reaches around 40%. After that, leverage declines mildly because households start to increase their holdings of risk-free non-housing assets, as can be seen from Figure 8.

Households save more risk-free financial assets as net worth increases, but decrease holdings of financial assets when their labor productivity is larger. The reason is that households with little wealth or higher productivity tend to finance their current and future consumption more from labor income. In contrast, high wealth and low productivity household tend to finance current and future consumption more from capital income. Thus high wealth and low productivity households have smaller shares of safe assets in their portfolio.

In the steady state, households buy houses, save low-interest bearing financial assets, and borrow through mortgages simultaneously. The reason is that households want to take advantage of the higher expected return from owning houses but also try to insure themselves against the adverse idiosyncratic depreciation shock. Since the real housing price is constant in the steady state equilibrium, uncertainty in housing returns only comes from the idiosyncratic depreciation shocks. When household is hit by large depreciation shock that puts their houses underwater, it defaults and its net worth only depends on how much financial asset

[18]With this definition, the household problem can be transformed into a consumption-savings problem which is available in the appendix.

[19]Given the expected housing depreciation is 1.2%, p equals 1.57 and p_s equals 0.043 in the steady state, the expected housing return is obviously higher than the risk-free interest rate which is equal to 1.2%.

it owns, $(1+r)a$. To smooth consumption, they find it optimal to hold risk-free financial asset, a.

In addition, when household chooses a larger leverage/mortgage debt given house h and net worth x, the housing down payment is smaller and household spends less in purchasing the house and thus can save a larger amount of financial assets. Since the financial assets are not seized by the banks when households default, accumulating financial assets enables them to reduce the fluctuations in consumption. Thus the benefit of borrowing through mortgages is the increase in household value from increasing and smoothing consumption with risk-free financial assets. On the other hand, the cost of borrowing is the reduction in household value due to larger default risk and net interest payments to the banks. Both the cost and benefit of borrowing are increasing in leverage. However, the cost of borrowing rises faster than the benefit given the format of the mortgage price schedule and the concavity of household value function. Thus there exists a unique optimal leverage policy for each given house h and housing prices p. The optimal leverage/mortgage debt is such that it equalizes the benefit and cost of borrowing in the residential mortgage market. Therefore, households would borrow through residential mortgages and save via financial and housing assets at the same time in the steady state.[20]

In the steady state, the model reproduces a housing foreclosure rate of 0.36% which is consistent with the data. Specifically, households that have their houses foreclosed on are mostly those with little net worth, because they are the high leverage takers at each labor productivity level.

The model generally reproduces the U.S. wealth distribution. Wealth in the model is defined as household net worth. Diaz-Gimenez et al (1997) reported that the Gini coefficient of wealth is 0.78 in the 1992 SCF. The wealth Gini coefficient in the steady state of this model is 0.39, which is close to that in the data and stands among the rates produced in the literature: Jeske et al (2011) obtains a Gini coefficient 0.46 in their steady state. Iacoviello and Pavan (2013) obtains a Gini coefficient equals to 0.73 in their steady state with two discount factors and 0.53 with a single discount factor.

In the steady state, housing wealth constitutes 32% of total household wealth, which is consistent with 30% in the data from 1969Q1-2006Q4. Moreover, housing wealth is 1.1 times that of real GDP in the benchmark economy, which matches the corresponding data from 1969Q1 to 2006Q4. In the steady state, 99.3% of households own strictly positive housing assets and 50.1% of households own larger houses than the amount of housing services they actually consume. Since housing is perfectly divisible in the model, I regard

[20]Mian and Sufi (2011) has documented that borrowed funds based on home equity are used by households for increasing consumption.

the "percent of households with $h' > s$" as the best proxy to the homeownership rate in the model.[21] Accordingly, the homeownership rate in the benchmark economy is close to the homeownership rate in U.S. data, which is 64% on average from 1994 to 2007.

5 Negative productivity shock

In this section, I present the results for the benchmark economy with a persistent negative productivity shock. In the first period, productivity drops 2.7% in the housing and consumption good sectors and recovers gradually afterwards according to equation (21). The initial decline in productivity is determined such that the measured total factor productivity (TFP) in the economy decreases about 2.8% initially to be consistent with the decline of measured TFP in the data during the Great Recession.[22]

Figure 10-12 shows the transitional paths of the key variables. When the productivity shock hits the economy at $t = 1$, the marginal productivity of capital (MPK) falls which leads to an initial decline in real interest rate. The real wage rate falls at the shock as the marginal product of labor (MPL) declines.[23] Given that the inverse of the elasticity of intertemporal substitution σ is equal to 3.9, the substitution effect dominates and households supply less labor. Thus aggregate labor supply drops initially and then increases slowly as the wage rate recovers. As aggregate productivity increases over time, the wage rate and interest rate recovers gradually.

Households reduce their consumption as they receive lower labor and asset income. The decrease in the MPK results in the initial drop of investment. The aggregate capital stock depreciates gradually as investment falls down.

Real house prices jump up 0.17% initially at the shock. It then decreases to be 0.32% below the steady state house prices as aggregate productivity recovers over time.[24] By equation (41) in the appendix, real house prices depend on aggregate productivity z, real interest rate r, and the housing capital's share ν. Given that the consumption good sector is relatively capital intensive as $\alpha > \nu$, real house prices are negatively related to the interest rate and positively related to productivity z. Since the initial drop in the capital rental rate $r + \delta_k$ is larger than the initial decrease in productivity z, real house prices increase about

[21] Note that homeownership here does not correspond to the traditional concept of owner-occupation. In this paper homeownership means that the households' holdings of housing assets can fully satisfy their own demand for housing services. This definition of homeownership is consistent with Henderson and Ionnides (1983). Under the assumption that the housing asset is perfectly divisible, this definition is the best proxy to the owner-occupation homeownership in the data.

[22] See Figure 2 and one can find that the Solow Residual decreased 2.8% during the recent financial crisis.

[23] MPK_c is equal to MPK_h as capital is perfectly mobile. Similarly, MPL_c and MPL_h are equal because labor is perfectly mobile across sectors.

[24] The decline in real house prices are relatively small compares to the size of the shock because aggregate productivity falls equally in consumption good and housing good sectors. If the shock took place only in the consumption good sector, real house prices would drop about 2% in the first period. However, housing output would increase initially in that case.

0.17% in the first period. Intuitively, real house prices are equal to the unit cost of housing production which hinges on productivity and factor prices as shown in equation (10) with the CRS production function. Even if factor prices w and r decline at the shock and reduce the production cost given the same K_h and N_h inputs, the smaller productivity reduces housing output so much that the unit cost of production still becomes higher. Therefore, real house prices rise at the shock as the unit cost of production is larger when productivity z falls.

Notice that the movements of the real house prices hinge on the capital's share in the housing sector, ν. In particular, if $\nu = 0$ so that housing sector only has labor as input, house prices p only depend on productivity z and wage w and not directly on the interest rate r. In this case, the movement of p in $t = 1$ depends on the ratio $\frac{w}{z}$. [25]

Aggregate net saving drops with the arrival of the shock. Specifically, it is households with high productivity and low wealth that experience the largest decline in net saving. The high productivity and low wealth people work more hours in the steady state, so the decline in wage and labor hours decreases their labor income greatly. To smooth consumption, they have to significantly reduce their net savings. In contrast, people with low labor productivity and high net worth supply very little labor in the steady state. As a result, the big drop in wage has little impact over their labor income. Given that the real interest rate recovers quickly after the initial decline, this group of households actually reduces their consumption and increases their net savings for the higher return next period. Since most of the population is in the first group, aggregate net saving declines with the arrival of the shock. Aggregate net saving declines further after the first period because households need to smooth consumption but labor and asset income only recovers slowly.

Figure 11 shows the dynamics of aggregate leverage. Recall this is defined as aggregate mortgage debt divided by the aggregate housing wealth. Aggregate leverage rises by 7.7% at arrival of the shock and falls gradually following period 2. The movements of individual leverage policy are consistent with the ups and downs of the aggregate leverage. As shown in Figure 12, the household leverage policy curve shifts up in period 1. To understand the change in leverage policy, suppose a household with the highest labor productivity is at point A in Figure 12 in the steady state. When the shock hits, the lower risk-free interest rate moves the household up to point B which corresponds to a higher leverage assuming his net saving does not change.[26] Nevertheless, the lower wage and interest rate reduce his labor income and asset income. To smooth consumption, he has to decrease net savings. Thus

[25] If instead the housing sector is more capital intensive such that $\alpha < \nu$, then real house prices are negatively related with productivity and positively related with real interest rate according to equation (41) in the appendix. Particularly, if $\nu = 1$ so that housing sector produces with capital only, house prices p are independent of w and only hinge on z and r. In this case, the initial movement of p depends on $\frac{r+\delta_k}{z}$.

[26] Notice that household labor productivity is persistent. So there is large probability for an individual to stay with his current labor productivity next period.

on the graph he moves up further from point B to point C which corresponds to a smaller saving and even higher leverage. Therefore, leverage at the individual level rises initially and there is no household deleverage process in the pure negative productivity shock.

Leverage falls after period 2 for two reasons. First, real house prices fall after period 2. The lower housing return discourages households from borrowing and purchasing houses.[27]. Second, as the real interest rate and wage rate recover over time, households does not need to borrow as much for their consumption and to purchase houses.

Aggregate financial assets rise initially upon the arrival of the shock and fall thereafter as productivity recovers. As explained in Section 4, households take on mortgage debt and also hold risk-free financial assets in order to smooth consumption because the idiosyncratic depreciation shocks reduces housing return and might trigger default. An increase in leverage ι implies a larger default risk as the threshold depreciation rate is equal to $1 - \iota$ and the default probability is equal to $1 - F(1 - \iota)$. Since all households take higher leverage when the shock hits, they insure themselves against higher default risk by holding more safe assets. Therefore, household demand for financial assets increases with the shock. As leverage falls when z recovers, households' holding of financial assets then decreases.

Aggregate housing demand declines about 0.13% upon the shock's arrival and falls to 1.5% below the steady state level in about 25 periods. Nevertheless, housing demand shows rich heterogeneity at the micro-level. Firstly, households with low wealth and high labor productivity decrease housing demand. They belong to the group of population that experience large decline in labor income. To smooth consumption, they have to decrease net savings. Since they finance their consumption more from labor income and save relatively less in risk-free assets in the steady state, housing wealth takes up a large share in their asset portfolio. Therefore, they reduce their holdings of housing assets as they have to save less. Secondly, households with low productivity and high wealth increase housing demand as they increase net saving. Since most people in the economy belongs to the first group, aggregate housing demand declines when the shock hits. Housing demand continues to decline for about 30 periods for two reasons. Firstly, household labor income recovers only slowly over time. Secondly, the return to housing investment decreases as real house prices fall after period 2.

Aggregate housing services decline when the shock hits because the reduction in the labor income leads to a large initial decline in total household consumption expenditure.[28] Therefore, the housing service expenditure declines as it takes up a fixed share (85.6%) in

[27] Although the rental price p_s increases after its initial decline, the effective housing price $(p - p_s)$ still falls from $t = 2$.

[28] Aggregate housing service is equal to aggregate housing demand H' according to the housing rental market clearing condition equation (13).

total household consumption expenditure. Aggregate demand for housing service declines further after period 1 as the rental price increases faster than the recovery of housing service expenditure.

Since capital and labor are perfectly mobile, the allocation of capital and labor inputs between the housing and consumption good sectors depends on the factor price ratio $\frac{w}{r+\delta_k}$, aggregate capital $\bar{K}$ and aggregate labor $\bar{N}$ in the economy.[29] At $t = 1$, the aggregate capital stock does not change, but aggregate labor falls 1.0% and the wage-rentals ratio $\frac{w}{r+\delta_k}$ increases about 0.93% so that the relative aggregate cost of labor falls. Thus, capital in the housing sector K_h turns out to decrease 2.7% in period 1. Since the consumption good sector is relatively capital intensive as $\alpha > \nu$ and the relative aggregate labor cost $\frac{w}{r+\delta_k}\bar{N}$ changes small, the decrease in the capital stock determines the dynamics of consumption capital as any change in aggregate capital is multiplied by $\frac{1-\nu}{\nu}$ and thus has a large impact over capital in the consumption sector. Therefore, K_c follows the pattern of aggregate capital by decreasing for about 25 periods until investment recovers above the steady state level. Capital in the housing sector continues to fall after the first period because the relative aggregate labor cost is decreasing faster than the impact from capital depreciation.

Labor input in the consumption good sector decreases 0.84% initially. This is because the factor price ratio $\frac{w}{r+\delta_k}$ rises at the shock and increases the capital-labor ratio in the consumption sector. Given that consumption capital only increases 0.08% at the shock, the consumption labor input turns out to decrease. The initial increase in the factor price ratio also increases the capital-labor ratio in the housing sector $\frac{K_h}{N_h}$.[30] Since capital in the housing sector drops at the shock, labor in the housing sector falls down 3.58% initially. As the factor price ratio decreases after the first period, the capital to labor ratio in the consumption and housing good sector follows to decrease. Thus labor input in the consumption sector recovers after period 2. As the housing capital to labor ratio decreases slowly over time, the fall in the wage-rental ratio is dampened by the small capital's share in the housing sector. Thus the housing labor input decreases with its capital input after the first period.

Housing investment, I_h, decreases 6.08% initially due to the declines in aggregate productivity and production inputs. Although productivity recovers after the initial decline, housing output continues to decrease for several periods as housing labor and capital inputs decrease further after $t = 1$. Thus housing production hits the trough six periods after the shock and the housing recovery thus comes later than the recovery of the consumption good sector.

[29]See appendix B.4

[30]The same increase in the factor price ratio would raise the capital-labor ratio in the housing sector more than that in the consumption sector as $\nu < \alpha$.

The foreclosure rate falls 0.14% in the first period when the productivity shock hits because there is an initial increase in real house prices. However, the foreclosure rate jumps to 6.83% above its steady state level in the second period as households choose larger leverage at $t = 1$ because the lower labor and asset income make them poorer. Foreclosure rate comes down gradually beginning in period 3 as household reduce leverage when labor and asset incomes recover.[31]

In summary, a persistent negative productivity shock itself can generate a recession, declines of housing demand and production, but it fails to generate decreases in aggregate financial assets, mortgage debts and leverage. To explain the fluctuations of mortgage debts and leverage in the data, I raise bank's cost of issuing mortgage to create a financial environment similar to that in the 2007Q3-2009Q2 financial crisis.

6 The Great Recession Experiment

Economic booms and busts are closely related to changes in housing financial conditions as the Great Recession is widely believed to be connected with the financial innovations in the mortgage market. The housing finance condition has been tightened in the financial crisis as a large fraction of financial institutions raises their down payment requirements and the issuance of the mortgage backed securities have been restricted.

To mimic the environment in the Great Recession, I raise banks' cost of issuing mortgage permanently, and at the same time keep the same productivity shock as that in Section 5.[32] The permanent change in housing finance and the persistent productivity shock together causes an economic transition until the economy reaches the new steady state. Specifically, the mortgage administration fee r_w is raised permanently from $1.0e - 4$ in the benchmark economy to $3.5e - 4$ in period 1.[33] The increase in r_w captures the increased cost of financial intermediation and a permanent structural change in mortgage finance as it results in higher housing down payments in the financial market. I call this two-shock experiment the Great Recession (G.R.) Experiment.

6.1 Transitional Dynamics of the Great Recession Experiment

When the negative financial and productivity shocks hit the economy, the marginal productivity of capital (MPK) and the marginal productivity of labor (MPL) both decrease. Thus

[31] Suppose the banking imbalance caused by the fluctuation of foreclosure rate at the shock is eliminated by the government funding which is borrowed from other countries outside the economy.

[32] Again aggregate productivity falls 2.7% initially and recovers gradually over time according to equation (21).

[33] I raise r_w to $3.5e - 4$ so that in the new steady state household leverage declines 17% to match the decrease of households leverage observed in the Great Recession shown in Figure 1.

real interest rate and wage decline initially. Since $\sigma = 3.9$ the substitution effect dominates and aggregate labor falls at the time of the shock. The reductions in aggregate productivity and labor input in the consumption sector together contribute to a 3.02% initial decline in real consumption good output. The housing output experiences an 9.91% initial decline as its productivity, labor and capital inputs decrease at the arrival of the shock.[34] The contractions in the consumption and housing output together contribute to an initial fall of 3.47% in aggregate output. As aggregate productivity increases, aggregate output, real wage and interest rate recover little by little over time. Comparing the impulse response of macroeconomic variables of the G.R. experiment to that in the pure productivity shock, I find that the decrease of aggregate productivity is responsible for the contractions of real aggregate output, consumption, investment and most of the housing production.[35]

Before discussing the transitional dynamics of the housing market, it is worth noticing that the initial decrease in the real interest rate is smaller in the GR experiment than its initial decline in the pure productivity shock. This is because the tighter financial condition raises the cost of borrowing so that households reduce leverage and take smaller mortgage debts. Accordingly, they need to save less financial assets to smooth consumption. The reduced saving in financial assets leads to smaller supply of credit in the financial market and thus there is less rental capital available for production.[36] Consequently, interest rate increases for households to deposit a larger amount of financial assets. Therefore, the initial decline in real interest rate is smaller in the G.R. experiment than that in the pure productivity shock.

The impulse responses of housing variables differ from that in a pure productivity shock in several dimensions. First of all, housing price increases initially but the increment is smaller.[37] According to equation (41) in the appendix, real house prices are positively related with productivity and negatively with the interest rate when $\alpha > \nu$. Since the productivity paths are the same but the initial decrease in interest rate is smaller in the G.R. experiment, real house prices rise by less upon the arrival of the shocks. The real house prices start to decrease at and after period 2 as the rental rate $r + \delta_k$ grows faster than aggregate productivity.

Secondly, aggregate leverage/mortgage debt slumps when the financial transition is trig-

[34] The decreases in N_h, K_h are explained in the later context.

[35] I have done an experiment with the tighter financial condition and no productivity shock and find that aggregate output falls less than 0.01%, consumption declines 0.01%, investment increases 1.4%, consumption good output rises up 0.1% and housing output drops 1.5% at the shock.

[36] See credit market resource constraint equation (15).

[37] The average quarterly decline of real house prices varies from 0.5% to 0.9% in all post-war recessions except the 2000-2001 recession and the Great Recession. The real house prices increased 1.3% on average each quarter in the 2000-2001 recession. The collapse of the housing price in the Great Recession might because that the pre-crisis housing price has severely deviated from the fundamental, i.e. the high housing price before the crisis is a bubble. Since this paper does not generate a price bubble in the steady state, it is reasonable that real housing price does not experience a big slump in this two-shock experiment. See footnote 23.

gered. As shown in Figure 14, aggregate leverage falls 8.40% at the shock and continues to decline to 17.83% below its steady state level in 25 periods. Consistent with the movements of aggregate leverage, leverage at the household level also falls when the financial transition starts. As shown in Figure 15, the household leverage policy curve shifts down greatly in period 1. To understand the shift of the leverage policy, suppose that a household with the highest labor productivity and high net worth is at point A' in the steady state. The tighter financial condition makes him move down to point B' which corresponds to a lower leverage, assuming his net saving does not change. Nevertheless, the lower wage rate decreases his labor income. To smooth consumption, he reduces net saving. Thus on the graph he moves from point B' to point C', which corresponds to less savings and about the same level of leverage as B'. Nevertheless, for households with the highest productivity and very small net worth, their leverage policy curve remains basically unchanged. However, they experience the largest decrease in labor income which leads them to reduce net saving significantly. Thus they end up taking larger leverage than in the steady state. Since few households are in the second group that increases leverage, the aggregate economy experiences a deleveraging process in the Great Recession Experiment.

Households deleverage because larger interest payments dampen households' incentive of borrowing mortgages to save financial assets and smooth consumption. Since households borrow at the mortgage interest rate and receive interest from banks via saving financial assets, the parameter r_w determines how much households are willing to borrow and how much risk-free financial assets they save.[38] On the one hand, the benefit of borrowing through residential mortgages is the increase in household value from smoothing consumption with larger holdings of financial assets. Borrowing larger debts allows them to spend smaller amounts out of their savings for the same houses and thus they can save more in financial assets. Since the risk-free financial assets are not seized by the banks when households default, accumulating financial assets enables them to keep their consumption close to the level before default.

On the other hand, the cost of borrowing through mortgages is the net interest payments that households pay out to the banks. Given a house h' and a housing price p, the benefit and cost of borrowing depend on r_w. A higher r_w shifts up the cost of borrowing at each level of leverage as households have to make higher net interest payments to the bank for the same (m', h') choice. In the meantime, a higher r_w shifts down the benefit of borrowing at each level of leverage as households receive a smaller mortgage loan with the same (m', h') because the loan is now discounted more heavily with the higher r_w. Thus the optimal

[38] The risk-free interest rate $= r$ while the risk-free mortgage rate $= r + r_w$. So the difference between the mortgage rate and the risk-free saving rate $\geq r_w$.

leverage that equalizes the benefit and cost of borrowing becomes much smaller than the optimal leverage in the steady state. Alternatively, with a tighter financial condition that increases the difference between the borrowing interest rate and saving rate, households find the cost of borrowing to save increases significantly relative to its benefit for the same (m', h'). Therefore, households choose smaller leverage / mortgages. Consequently, the housing down payment has increased after the financial change has taken place. Thus, the tightening of the housing financial condition explains the sharp declines in aggregate leverage, aggregate mortgage debt and aggregate financial assets. This is the mechanism of the household deleveraging process.

Thirdly, aggregate housing demand decreases at the shock as in the pure productivity shock in Section 5, but the trough in this experiment is deeper. Households choose smaller leverage and a higher down payment with financial tightening, but the large decline in mortgage debt does not follow with a great decline in aggregate housing demand because households is spending more of their own “skins” on the purchase of houses. Nevertheless, the smaller borrowing makes housing demand fall by more than in the pure productivity shock experiment.

The foreclosure rate drops 0.1% in the first period because housing value appreciates as the real house prices rise. The foreclosure rate shows a large drop starting in period 2 as households deleverage once the financial tightening takes place. Since the financial change is permanent, foreclosure rate converges to a lower level in the new steady state with $r_w = 3.5e - 4$.

In the G.R. experiment, there are contractions in consumption good and housing good outputs. However, the contraction in the consumption good sector is smaller than that in the pure productivity shock while the contraction in the housing sector is larger. This is because the real interest rate decreases less at $t = 1$ in this experiment. A higher initial real interest rate implies a higher MPK and a smaller capital-labor ratio in both sectors at the shock. So the aggregate investment is larger as MPK is higher. The smaller capital-labor ratio implies a lower MPL and wage rate in the G.R. experiment at $t = 1$. Given a smaller wage, aggregate labor supply is thus smaller initially. Since aggregate labor $\bar{N}$ falls by 1.12% and the wage-capital rental ratio only increases by 0.74% at $t = 1$, capital in the housing sector decreases by about 6.8% which is much larger than the initial drop of K_h in the pure productivity shock case. In the meantime, K_c increases 0.19% with the arrival of the shock compare to the tiny initial decrease in the pure productivity shock. On the other hand, labor input in the consumption good sector falls less while labor in the housing sector falls much more initially. Therefore, the decrease in housing output is larger than that in the pure productivity shock while the decline in consumption output is smaller.

As in the pure productivity shock case, capital in the consumption good sector follows the pattern of aggregate capital, decreasing for about 25 periods. In contrast, capital in the housing sector starts to increase after the initial decline as aggregate capital depreciates and the relative cost of aggregate labor recovers over time. As the factor price ratio falls after the first period, the capital-labor ratio decreases further and it follows that labor inputs in both sectors recover fast from period 2. From period 2, housing output increases little by little as productivity, housing capital and labor recover.

Comparing the movements of model variables to the fluctuations of corresponding variables in the data, this two-shock experiment is able to explain about 70% of the drop in real output, 38% of the reduction in consumption, 12% of the decline in labor hours, the entire drop in mortgage debts, 22% of the decrease in housing output, 22% of the decrease in housing demand and 1.2% of the decline in real house prices.

6.2 The new steady state with tighter housing finance

This subsection discusses the properties of the economy in the new steady state with tighter housing finance condition ($r_w = 3.5e - 4$). As shown in table 4, with the tighter housing finance condition the quantities of most real economic variables such as aggregate output, consumption, investment, housing demand are basically the same as that in the benchmark economy. On the production side, variables such as capital inputs, labor inputs and outputs in both sectors barely change with the tighter finance condition. In addition, the equilibrium real wage, real interest rate and real rental price also change little. Since interest rate only decrease 0.5 basis points, the risk-free mortgage interest rate increases 2 basis points in the new steady state. The plunge of the outstanding mortgage debts does not conflict with the lower risk-free mortgage interest rate because it is the difference between the risk-free interest rate and the mortgage interest rate rather than the absolute cost of borrowing that matters critically in household borrowing and saving decisions.

Several major changes take place in the housing market. With the tighter housing finance condition, aggregate mortgage debt falls to around 80.3% of its benchmark value, foreclosure rate drops 13.9%, and aggregate financial asset falls 3.33%. Aggregate housing stock is 0.24% smaller than the benchmark level but homeownership rate, which is represented by the percent of households with $h' > s$, increases 0.26%. If I use the percent of households with $h > 0$ to characterize homeownership rate, homeownership rate does not change with the tighter housing finance. Non-housing asset now takes up 67.48% of household wealth, which is smaller compares to 68.17% in benchmark economy. In addition, tighter housing finance leads to a small increase in wealth inequality as the Gini coefficient is 0.3% higher than that in the benchmark economy. Finally, the tighter borrowing in the mortgage market leads to large increase in home equity as shown in Figure 16. The increase in home equity implies that the housing down payment in the financial market is higher in the new steady state.

Comparing the new steady state with tighter mortgage issuance to the benchmark economy, I can draw a couple of policy implications. If the government values higher homeownership and a smaller foreclosure rate for the stability of the financial system, they should adopt regulations to enforce tighter mortgage issuance. If the government cares more about inequality, they should subsidize the mortgage issuance sector to reduce the extra cost measured by r_w so that the cost of borrowing by means of mortgages is smaller and pushes the economy to move toward the benchmark case.

7 The Foreclosure Experiment

The Great Recession Experiment can explain over the macroeconomic recession and housing recession except the foreclosure rate does not increase initially as in the data. Therefore, I add a third shock – a one-period housing depreciation shock to the G.R. experiment. Specifically, the housing depreciation shock is a one period shock that raises the shape parameter γ in the Pareto distribution for one period such that the housing foreclosure rate would increase to 0.73% in the first period to match the 200% increase of mortgage foreclosure rate in 2007Q4 relative to the average foreclosure rate from 2002Q1 to 2006Q4. The three shocks are unexpected by the households and they hit the economy simultaneously in period one. I call this three shock experiment the Foreclosure Boom (F.B.) Experiment.[39]

When the shock hits, a larger fraction of households wake up at $t = 1$ to see their housing values fall below the mortgage loan value and thus choose to default. The high housing depreciation rate decreases household net worth and brings a large negative wealth effect to the economy. The results of this experiment differ from that in the G.R. experiment in several respects. First of all, with the same tightening in the housing finance condition, the initial decline in mortgage loan, leverage and financial assets is smaller than that in the G.R. experiment. This is because households are poorer as net worth shrinks so that they need to borrow more per unit of housing asset they purchase. Given that they choose a larger leverage, the initial decrease in the aggregate financial assets is smaller as they need a larger amount of safe financial asset to insure themselves against the higher default risk.

[39]Notice that the Pareto distribution does not change in the mortgage price schedule as all households and banks in the economy expect that the increase in γ only lasts for one period.

Secondly, households have a larger initial decrease in consumption, net saving and housing demand. Compare to households in the G.R. experiment, they reduce their consumption and net saving more as they are poorer. Households still exhibits similar heterogeneity in net saving, but the heterogeneity of housing demand that we observe in the G.R. experiment does not apply to the F.B. experiment. Instead, all households decrease their housing demand as they become poorer.[40] Accordingly, housing demand declines 0.88% initially.

Thirdly, the real interest rate, investment, and aggregate capital have larger declines in the Foreclosure Experiment. The real interest rate falls more than that in the G.R. experiment for two reasons. The initial decrease in financial assets is smaller in this experiment so that the credit supply is larger. Also, due to the strong negative wealth effect from the depreciation shock, households work more hours than they would in the G.R. experiment so that the initial decrease in aggregate labor is smaller. The higher aggregate labor increases $\frac{w}{r+\delta_k}$ and thus increases the capital-labor ratio in the consumption and housing sectors. Thus the MPK and the real interest rate are smaller. With the lower MPK, investment is also smaller in this experiment. Consequently, aggregate capital depreciates to hit a lower trough.

Fourthly, the recession in the consumption sector is deeper when the shock hits and the recovery in the housing sector is later than that in the consumption good sector. Since labor and capital are perfectly mobile across sectors, by equation (39) and (40), higher aggregate labor increases the capital and labor inputs in the housing sector and reduces capital and labor in the consumption sectors when the shock hits given that the housing sector is relatively labor intensive. As a result, capital and labor in the consumption sector decreases more than they do in the G.R. experiment. The smaller capital and labor inputs make consumption output decrease 3.57%, which is larger than that in the G.R. experiment.

In contrast, housing output, I_h, only decreases 0.60% at the shock. This is because the higher factor price ratio makes the housing capital input increase about 3.19% initially. As aggregate productivity recovers, $\frac{w}{r+\delta_k}$ is falling fast so that the relative cost of aggregate labor $\frac{w}{r+\delta_k}\bar{N}$ falls more than the decrease in $\bar{K}$. Therefore, the capital input in the housing sector falls further after $t = 1$. The housing labor input also continues to decrease after period one. Although productivity recovers after its initial decline, housing output continues to fall for several periods as housing labor and capital inputs decrease further after the first period. Therefore, the contraction in the housing production sector hits the trough seven periods after the shock and the housing recovery is later than the recovery in the consumption good sector.

[40] All households decrease their demand for mortgage loan, housing asset and financial asset in the F.B. experiment. However, these do not contradict with the fact that low productivity high wealth households increases their net saving slightly because $g' = a' + ph' - m'p_m(m', h')$. The increase in net saving is possible when the decrease in the value of mortgage loan is larger than the decreases in financial asset and housing asset. This is exactly what happens to the high wealth low productivity group.

Finally, the foreclosure rate hits 0.73% initially which is 102% higher than the steady state default rate. From the second period and on, the foreclosure rate falls as households deleverage with the tighter housing finance condition.

Real housing price increases 0.21% initially which is higher than that in the G.R. experiment because the higher initial wage has a dominating effect with the small ν and thus raises the unit cost of production. Consequently, the house prices are higher when the shock hits. Real house prices start to fall after the initial increase as productivity recovers gradually. This reduces the unit cost of housing production over time.

In a nutshell, the Foreclosure Boom experiment generates a real economic recession alongside a recession in the housing market. The contribution of the depreciation shock is to increase the foreclosure rate initially. Comparing the model declines to that in the data, the three-shock experiment is able to explain about 70% of the drop in real output, 39% of the reduction in consumption, 10% of the decline in labor hour, the entire drop in mortgage debts, 12% of the decrease in housing construction output, 22% of the decrease in housing demand and 1.3% of the decline in real house prices.

8 The Depreciation Shock Experiment

In this section, I discuss the aggregate economic effect of a single depreciation shock. Suppose the shape parameter γ in the Pareto distribution follows a log-AR(1) process

$$log\gamma_{t+1} = \rho_\gamma log\gamma_t + \xi_\gamma, \quad \xi_\gamma \sim N(\mu_\gamma, \sigma_\xi^2) \tag{23}$$

where $\rho_\gamma = 0.9$, $\sigma_\xi = 0.01$, $\mu_\gamma = \gamma^*(1-\rho_\gamma)$ and $\gamma^* = 0.7304$ as in the benchmark economy. When the shock hits, the shape parameter γ increases one standard deviation and then moves back to its steady state level according to the log-AR(1) process in equation (23).

The results of the depreciation shock are displayed in Figure 19-20. Firstly, households choose a smaller mortgage debt level and leverage upon the arrival of the shock. However, the reason that households deleverage here is different from that in the G.R. experiment. Households reduce their leverage because the mortgage price schedule provided by the bank has changed with the Pareto distribution.[41] Holding household leverage decisions constant, a higher shape parameter γ means that households have larger probability of drawing big depreciation rates, increasing the probability that their houses goes underwater. Moreover, the housing value after the foreclosure process is also lower with the higher γ holding everything else constant. As such, banks offer a lower mortgage price for the same leverage

[41] Once the shock hits, households and financial institutions have perfect foresight over the path of the shape parameter γ.

ratio to compensate themselves for the increase in expected loss from higher default rate and the smaller after depreciation housing value in the foreclosure process. Given the new mortgage price schedule, the cost of borrowing using mortgages has been shifted up so that households find it optimal to choose a lower leverage ratio. Accordingly, their demand for financial assets also declines. As γ recovers, banks offer better and better mortgage prices to households so that they gradually raise their leverage and debt positions. Financial asset increases gradually as household leverage recovers over time.

Housing demand decreases at the shock for two reasons. Firstly, housing demand decreases because the increase in housing depreciation reduces the real housing return. Secondly, as households choose smaller leverage/mortgage debts, they pay higher housing down payments per unit of house they purchase. Given that their net worth decreases when the shock hits, they can afford to purchase less housing.

The persistent depreciation shock generates a deep contraction in the housing sector but is not able to generate a severe real economic recession. The initial decline in financial assets leads to the increase of the real interest rate at the shock's arrival as credit supply is smaller. An initial increase in the interest rate implies a lower capital-labor ratio by equation (46) and (48). Thus the MPK is higher while the MPL is lower in both sectors. The lower MPL explains the initial decrease in real wage. Given that the inverse of the elasticity of intertemporal substitution σ is equal to 3.9 and the real wage falls, the substitution effect dominates and households supply smaller labor hours. With lower aggregate labor and factor price ratio, capital and labor in the housing sector decrease greatly according to equation (56). Thus housing output decreases 15%. Since aggregate capital does not change initially, it follows that the consumption sector uses a slightly larger amount of capital as the housing sector is small and only takes up 6.8% of aggregate output. Higher capital increases the MPL in the consumption sector which makes consumption good firms employs more labor. Therefore, labor in the consumption sector increases by 1% initially. Since aggregate productivity remains at steady state level, the increase in consumption good inputs leads to a 0.8% initial increase in consumption output. Although the depreciation shock brings a small boom in consumption good production, real aggregate output still falls 0.1% because the increase in consumption output cannot make up for the large contraction in housing.

Although the real interest rate rises initially, the decrease in labor income and the small decrease in net worth make all households decrease their consumption demand and net saving slightly.

Real house prices decrease 0.1% initially as the decrease in real wage has a dominating effect so that the unit cost of production in the housing sector declines. As the wage recovers, real house prices also increase over time.

The foreclosure rate rises by 10% initially as a larger fraction of the population finds their housing assets underwater when the shock hits. The foreclosure rate falls at and after $t = 2$ for two reasons. Firstly, households deleverage in the first period and thus face smaller default risk. Secondly, γ starts to recover since period 2. The foreclosure rate becomes slightly higher in period 3 than period 2 as household leverage increases. Since period 4, the foreclosure rate moves down gradually as γ recovers to the steady state level gradually over time.

9 Concluding Remarks

This paper develops a dynamic stochastic general equilibrium (DSGE) model with heterogeneous households and two sectors to explore the impact of housing financial shocks, productivity shocks and housing depreciation shocks on the aggregate economy and on housing variables such as real mortgage debt and housing demand. I calibrate the model to reproduce the housing and non-housing moments in the U.S. data. The resulting economy is characterized by the household behavior of borrowing using mortgage debt and saving using risk-free financial assets and risky housing assets at the same time.

Several experiments have been conducted in order to understand the housing and real economic contractions during the Great Recession. Comparing the results in these experiments, I have come to the following conclusions. Firstly, the decrease of aggregate productivity accounts for the declines in aggregate output, consumption, business investment, labor hours as well as part of the drop in housing production and demand. Housing demand falls upon the decrease of productivity as households who experience large declines in their labor income hold less housing assets. Secondly, the tighter housing finance is responsible for the large decreases in mortgage debt, leverage and the part of the decrease in housing demand. Household decreases mortgage debt and leverage as the cost of borrowing mortgage to save financial assets have greatly increased. Under normal financial conditions, households take larger mortgage debts to take advantage of high housing return and insure themselves against high default risk by holding financial assets to smooth consumption. When the financial condition is tighter so that the cost of borrowing mortgages to save financial assets becomes higher while the benefit of borrowing decrease, households reduce leverage and their holdings of mortgage debts to avoid higher interest payments and default risk. Housing demand falls with tighter financial condition because household has to pay a higher housing down payment for the same house as they deleverage.

Thirdly, the persistent housing depreciation shock explains the movements of the foreclosure rate and part of the decreases in mortgage debt, housing demand and production. Housing demand is smaller with the depreciation shocks for two reasons. First, the higher depreciations decreases the real housing return. Second, the higher household default risk is priced by the financial intermediaries in the mortgage schedule when the depreciation process changes. Given lower mortgage prices in the financial markets, households take out smaller mortgages per unit of houses they purchase. Thus they decrease housing demand as the housing down payments are higher. Finally, the Foreclosure Boom Experiment with negative productivity shocks, financial tightening and housing depreciation shock generates a real economic recession alongside a housing contraction that explains the housing and non-housing variables apart from the movements of the real house prices.

Future research can still be strengthened in two respects. Firstly, one-period mortgage contract is modeled in the paper for the convenience of analysis. In the real world, most mortgage contracts are multi-period. The effects of the long-term mortgage debts to the macroeconomy and the housing market have not been fully explored in the literature. Secondly, this paper cannot explain the dynamics of the real house prices in the Great Recession. The fluctuations of real house prices in the model are not comparable to the collapse of the real house prices in the Great Recession. A potential way to fit the plunge of real house prices is to generate a housing price bubble in the steady state. These two issues are for further research.

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Appendices

A Data

I generate Figure 1 using the following data between 1969Q1 and 2013Q1. Housing wealth is the market value of the household owner-occupied real estate including vacant land and mobile homes from the Flow of Funds Accounts of United States. Data on units of new residential construction started is from the U.S. Census Bureau between 1969M1 to 2013M3. I convert the monthly data into quarterly data by taking sum of the monthly data in a quarter. The house price index is from the Freddie Mac which is available between 1975M1 and 2013M3. I obtain the quarterly price index by taking average of the monthly data in a quarter. Mortgage debt is obtained from the Board of Governors historical database. Housing foreclosure rate is reported in the National Delinquency Survey from 2004 to 2012 by the Mortgage Banker Association. Leverage is measured using real mortgage debt divided by real housing wealth. All real variables are obtained using GDP deflator which is measured using Gross Domestic Product (billions of dollars) from Table 1.1.5 divided by Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the Bureau of Economic Analysis.

For the data that generates Figure 2, output is the Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the BEA. Consumption is Personal Consumption Expenditures less durable goods from Table 1.1.5 of the BEA. Business investment is the sum of durable goods and private nonresidential fixed investment from Table 1.1.5. Residential investment is also from Table 1.1.5. The real values of consumption and investment are calculated using the GDP deflator. Real GDP, consumption, business investment and residential investment are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1969Q1 to 2013Q1. Data on unemployment rate is from the Bureau of Labor Statistics. The Solow Residual is measured using data on private capital and hours. Private capital is the sum of private fixed assets and consumer durables from Fixed Asset Table 1.1 of the BEA. Hours data are those constructed by Cociuba, Prescott and Ueberfeldt (2012).

B Solution Methods

B.1 Consumption-Leisure-Savings Problem

Following Jeske, Krueger and Mitman (2011), household's problem in this model can be transformed into the following consumption-leisure-savings problem

$$v(x,\epsilon) = \max_{c,g',n} u(c, 1-n; p_s) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon)\omega(g',\epsilon') \tag{24}$$

subject to

$$c + g' = w\epsilon n + x \tag{25}$$

$$0 \leq n < 1, \quad g' \geq 0 \tag{26}$$

where $\omega(g',\epsilon')$ is given by

$$\omega(g',\epsilon') = \max_{h',m',a' \geq 0} \int_{\underline{\delta}}^{1} v(x',\epsilon')dF(\delta') \tag{27}$$

s.t.

$$x' = (1+r')a' + \max\{0, p'(1-\delta')h' - m'\} \tag{28}$$

$$g' = a' + (p - p_s)h' - m'p_m(m',h') \tag{29}$$

In order to solve the consumption-savings problem, I need to solve $\omega(g',\epsilon')$ first. I call this optimization problem that solves $\omega(g',\epsilon')$ the household portfolio problem. The portfolio problem is represented by equations (26)-(28) which involves selecting houses, financial assets and mortgage debt to maximize expected value.

B.2 The Steady State

1. Building grid points for individual labor productivity, net worth x and net saving g. I use 23 log-spaced points over $[0, 45]$ for g, 33 log-spaced points over $[0, 45]$ for x. The result of the model does not change much with a more intensive grid. I approximate the continuous AR(1) process for labor productivity with a 5 state Markov chain using the procedure in Tauchen and Hussey (1991).

2. Set the guesses of real interest rate r and real rental price p_s.

3. Given r and p_s, do the following to find the value function and optimal decision rules:

(i) Set a guess for value function $v(x,\epsilon)$

(ii) Solve the household portfolio problem and find $\omega(g,\epsilon)$ for each (g,ϵ) on the grid.

(iii) Solve the consumption-leisure-savings problem represented by equation (24)-(26) to update value function and find the optimal net saving g^* for each (x, ϵ) on the grid.

(iv) Given $g^*(x, \epsilon)$, find the optimal asset decision rules $h^*(x, \epsilon)$, $m^*(x, \epsilon)$ and $a^*(x, \epsilon)$ using the decision rules solved in (ii).

4. Compute the steady state distribution of individuals over net worth, idiosyncratic productivity and default choice $\mu(x, \epsilon, d)$.

(i) I use 750 evenly spaced grid points over $[0, 45]$ for x and the same grid points chosen in step 1 for ϵ.

(ii) Approximate the housing, asset, mortgage decision rules solved in Step (3) using splines. For households starting with (x, ϵ, d), use the decision rules to determine the cutoff depreciation rate $\delta^* = 1 - \frac{m'}{p'h'}$. Then $1 - F(\delta^*)$ fraction out of $\mu(x, \epsilon, d)$ defaults and $F(\delta^*)$ fraction repay.

(iii) For people starting with (x, ϵ, d), they have the same δ^* and decision rules, but they might end up with different net worth next period since housing depreciation δ draw is idiosyncratic. I simulate δ draws for households at (x, ϵ, d) using Pareto distribution and calculate their future net worth x' correspondingly.

(iv) Suppose future net worth is such that $x_j \leq x' \leq x_{j+1}$ for a individual at (x, ϵ, d), where x_j and x_{j+1} are two adjacent net worth grid points. Then update μ assuming that individuals at (x, ϵ, d) move to (x_j, ϵ', d) with probability $\varphi \pi_\epsilon(\epsilon'|\epsilon)$ and to (x_{j+1}, ϵ', d) with probability $(1 - \varphi)\pi_\epsilon(\epsilon'|\epsilon)$, where $\varphi = \frac{x_{j+1} - x}{x_{j+1} - x_j}$. Update μ until it converges. The resulting μ is the fixed point of household distribution $\bar{\mu}$

5. With $\bar{\mu}$, calculate aggregates in the economy and solve the production problems. Then check market clearing conditions. If rental market and consumption good market do not clear, update guesses of r and p_s and come back to step 3.

B.3 Transitional Dynamics

The transitional dynamics is solved following three steps:

(1) Start with steady state value function and solve the value function backwardly from $t = T - 1$ to $t = 1$

(2) Start with steady state distribution, update the household distribution forwardly from $t = 1$ to $t = T$

(3) Check the market clearing conditions in each period to update interest rate and rental price.

The length of time $T = 160$ in the negative productivity shock and $T = 270$ in the Great Recession Experiment and the Foreclosure Experiment. $T = 120$ in the depreciation shock experiment. Since households in this model are forward looking, they look at current as well as future prices to make decisions. When the period t prices are updated, they have impact over the markets in period $t-1$ and t. Therefore, I use a small parameter 0.001 to update prices. The precision for value function $= 1.0e-4$. Precision for household distribution $= 1.0e-8$. Precision for market clearing conditons in the steady state and the transitional dynamics $= 1.0e-3$. All other precisions $= 1.0e-6$.

B.4 Solving the Production Problems

First order conditions for consumption good sector:

$$[K_c] \quad \alpha z K_c^{\alpha-1} N_c^{1-\alpha} = r + \delta_k \tag{30}$$

$$[N_c] \quad (1-\alpha) z K_c^{\alpha} N_c^{-\alpha} = w \tag{31}$$

First order conditions for housing sector:

$$[N_h] \quad (1-\nu) p z K_h^{\nu} N_h^{-\nu} = w \tag{32}$$

$$[K_h] \quad \nu p z K_h^{\nu-1} N_h^{1-\nu} = r + \delta_k \tag{33}$$

Equation (30) and (31) imply

$$w = \left(\frac{1-\alpha}{\alpha}\right) \left[\alpha z \left(\frac{1}{r+\delta_k}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{34}$$

$$\frac{K_c}{N_c} = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{w}{r+\delta_k}\right) \tag{35}$$

Equation (32) and (33) imply

$$p = \frac{1}{z\nu}\left(\frac{\nu}{1-\nu}\right)^{1-\nu} (r+\delta_k)^{\nu} w^{1-\nu} \tag{36}$$

$$\frac{K_h}{N_h} = \left(\frac{\nu}{1-\nu}\right)\left(\frac{w}{r+\delta_k}\right) \tag{37}$$

Recall the labor market, capital market clearing conditions

$$K_c + K_h = \bar{K}, \quad N_c + N_h = \bar{N} \tag{38}$$

Together can solve

$$K_c = \frac{\left(\frac{1-\nu}{\nu}\right)\bar{K} - \left(\frac{w}{r+\delta_k}\right)\bar{N}}{\frac{1-\nu}{\nu} - \frac{1-\alpha}{\alpha}} \tag{39}$$

$$K_h = \frac{\left(\frac{w}{r+\delta_k}\right)\bar{N} - \left(\frac{1-\alpha}{\alpha}\right)\bar{K}}{\frac{1-\nu}{\nu} - \frac{1-\alpha}{\alpha}} \tag{40}$$

Using equation (34) and (36) can obtain

$$p = \frac{1}{\nu}\left(\frac{\nu}{1-\nu}\right)^{1-\nu}\left(\frac{1-\alpha}{\alpha}\right)^{1-\nu}(\alpha)^{\frac{1-\nu}{1-\alpha}}\left(\frac{z}{r+\delta_k}\right)^{\frac{\alpha-\nu}{1-\alpha}} \tag{41}$$

$$\frac{w}{r+\delta_k} = \alpha^{\frac{1}{1-\alpha}}\left(\frac{1}{1-\alpha}\right)\left(\frac{z}{r+\delta_k}\right)^{\frac{1}{\alpha}} \tag{42}$$

	Interpretation	Value	Source
δ_k	capital depreciation	0.025	U.S. data
ρ_ϵ	productivity persistence	0.98	Jeske et al (2011)
σ_ϵ	productivity variance	0.30	Jeske et al (2011)
ν	capital's share in housing	0.12	GDP-by-Industry
θ	foreclosure technology	0.78	Pennington and Cross (2004)
σ	CRRA parameter	3.9	Jeske et al (2011)

Table 1: Exogenously Adopted Parameters

	Target Moment	Model	Target	Data Source
β	risk-free rate	1.2	1.2	U.S. data
τ	Average labor hours	0.33	0.33	U.S. data
τ_1	Consumption's share	0.86	0.86	NIPA
r_w	Aggregate Leverage	0.39	0.41	Flow of Funds
α	Total Capital to Output ratio	3.07	3.00	Gervais (2002)
γ	Foreclosure rate	0.36%	0.4%	MBA(2006)
σ_δ	House value volatility	0.07	0.08	OFHEO HPI data
$\underline{\delta}$	Average housing depreciation	1.2%	1.4%	NIPA

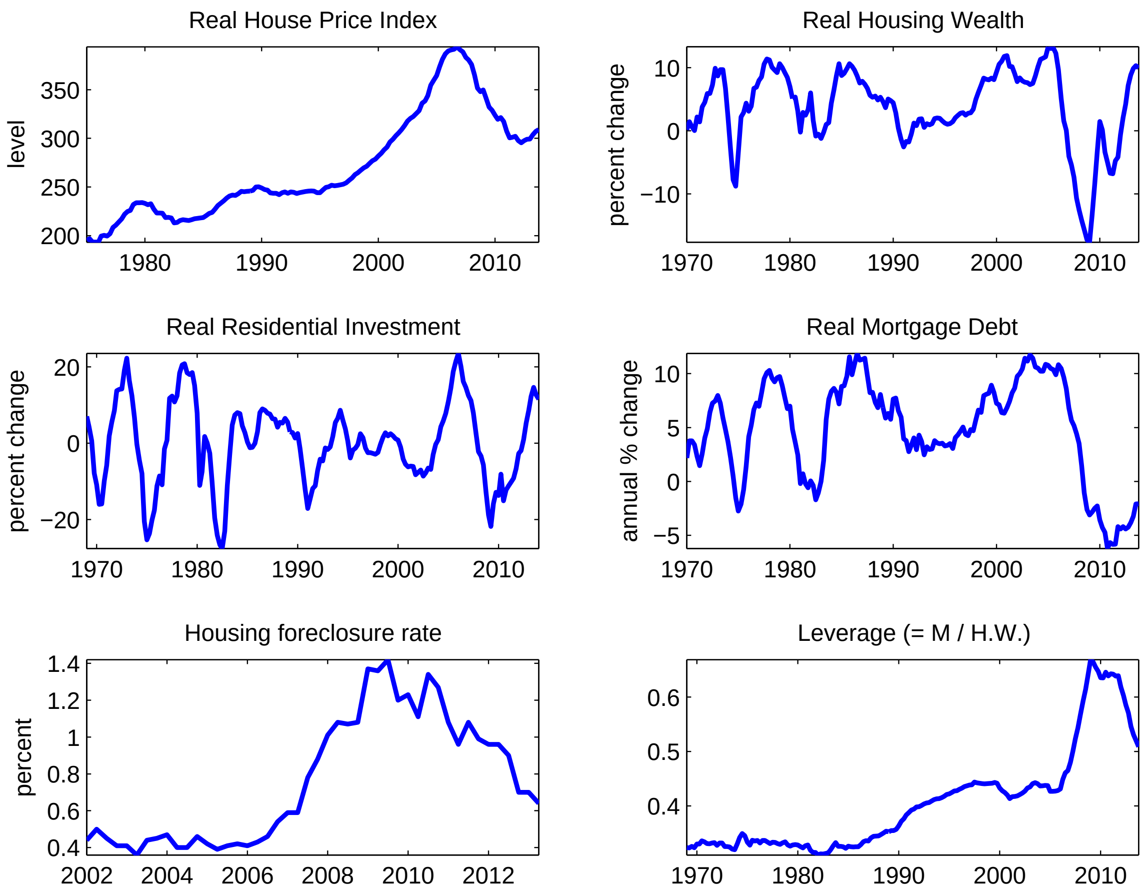
Table 2: Calibrated parameters and data moments

Variable	Interpretation	SS Value	U.S. Data
percent of hhs with $h' > 0$		99.3%	
percent of hhs with $h' > s$	homeownership	50.1%	64%
Wealth Gini	wealth inequality	0.39	0.78
$pH/(4 \times GDP)$	housing wealth	1.1	1.1
Non-housing asset	non-housing asset share	68.17%	70%
$p \times I_h/GDP$	housing investment share	6.6%	6%

Table 3: Steady State Numerical Results

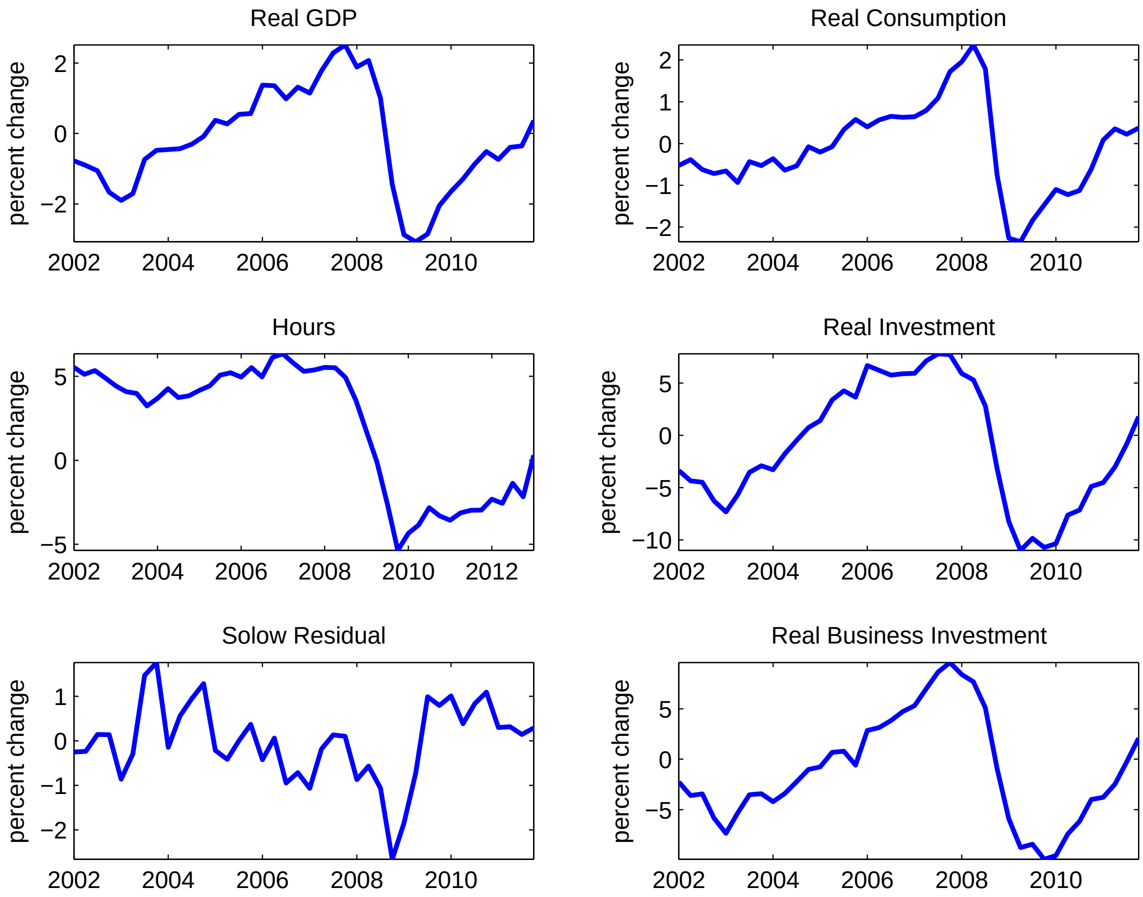
Variable	Interpretation	Benchmark value ($r_w = 0.0001$)	High financial cost ($r_w = 0.00035$)
r	real interest rate	1.196%	1.191%
w	real wage rate	1.725	1.726
p	real housing price	1.5695	1.5700
p_s	real rental price	0.0428	0.0429
A	financial asset	***11.10***	***10.73***
H	housing stock	3.302	3.294
Ml	mortgage loan	***1.971***	***1.583***
Default rate	foreclosure	***0.36%***	***0.31%***
Mean net worth	Mean net worth	1.80	1.80
C	consumption	0.861	0.861
Y	output	1.167	1.168
I	business investment	0.2286	0.2293
K	capital	9.129	9.149
N	effective labor	0.48	0.48
I_h	housing investment	0.049	0.049
$\mu(h' > 0)$		99.3%	99.3%
$\mu(h' > s)$	homeownership	***50.18%***	***50.31%***
Non-housing asset	non-housing asset share	***68.17%***	***67.48%***
Wealth Gini	wealth inequality	***0.392***	***0.393***
$pH/(4 \times GDP)$	housing wealth to output	1.11	1.11
pI_h/GDP	housing investment share	6.58%	6.59%

Table 4: Numerical Results of Higher Financial Intermediation Cost



Data Source: FHFA, Board of Governors, Flow of Funds, U.S. Census Bureau, M.B.A

Figure 1: Housing Market in the Great Recession



Data Source: NIPA tables, Fixed Asset tables and CPU (2012). Real GDP, consumption, business investment and residential investment are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1969Q1 to 2013Q1.

Figure 2: Other Macroeconomic Series in the Great Recession

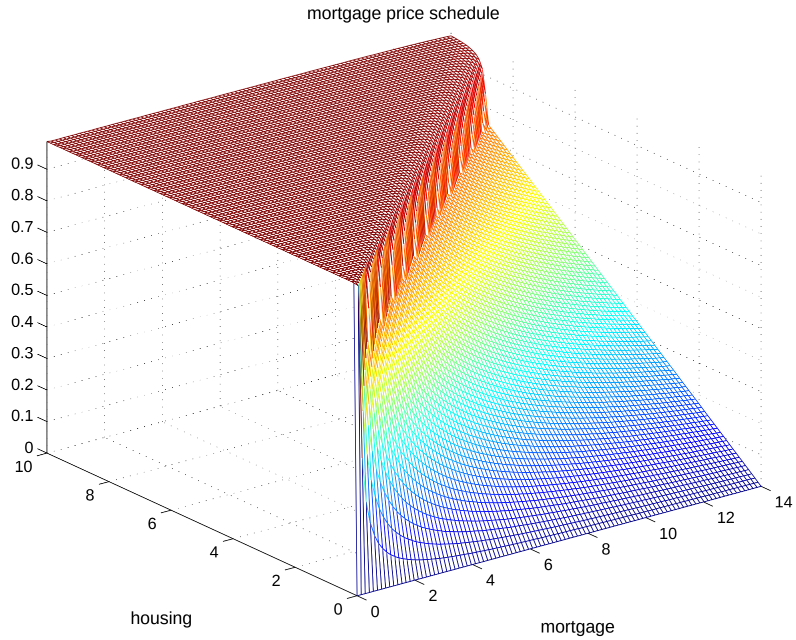


Figure 3: Mortgage Price Schedule

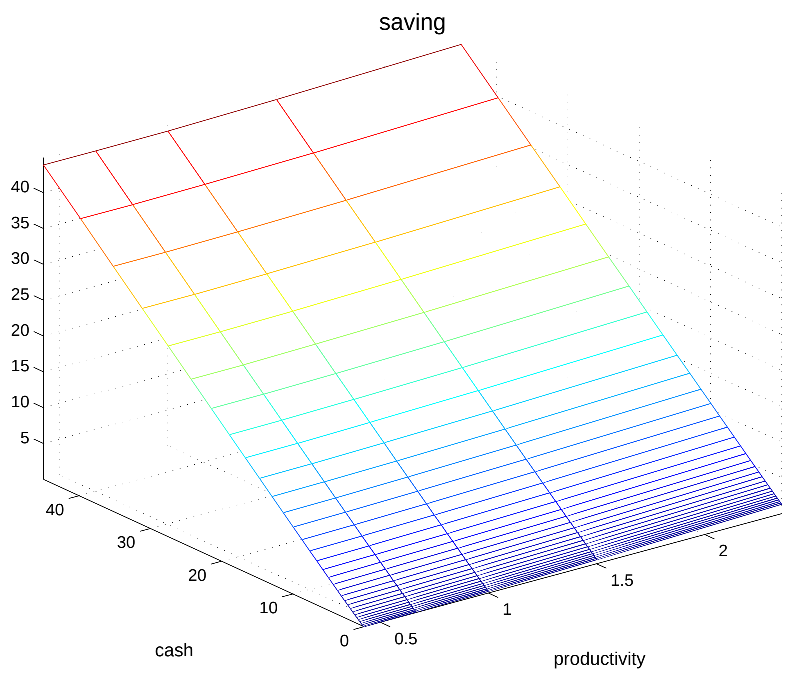


Figure 4: Net Saving Policy

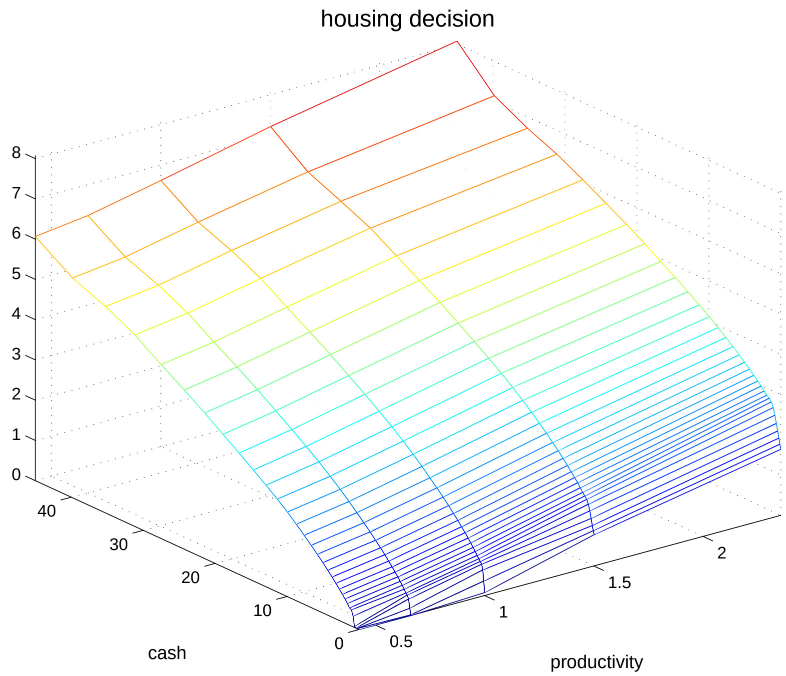


Figure 5: Housing Asset Policy Function

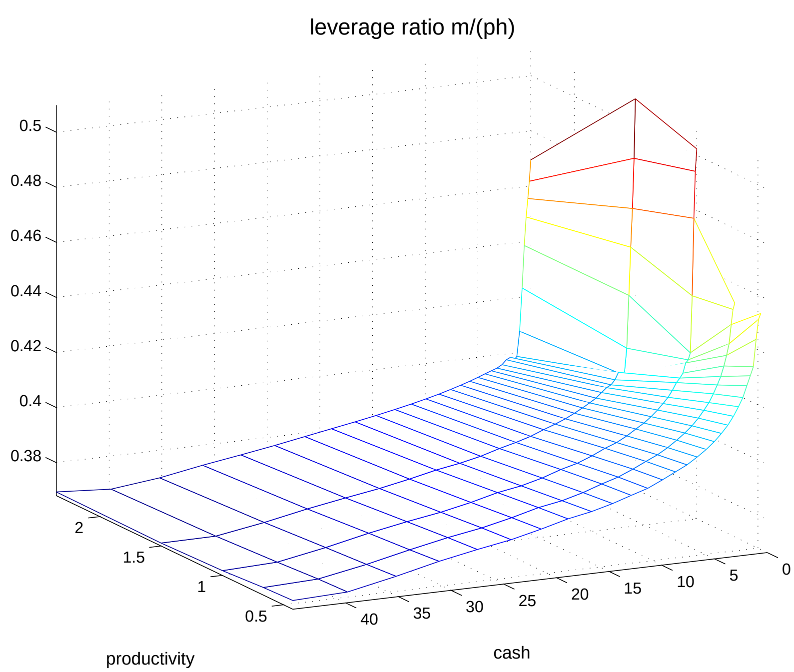


Figure 6: Household Leverage Policy, l=m/(ph)

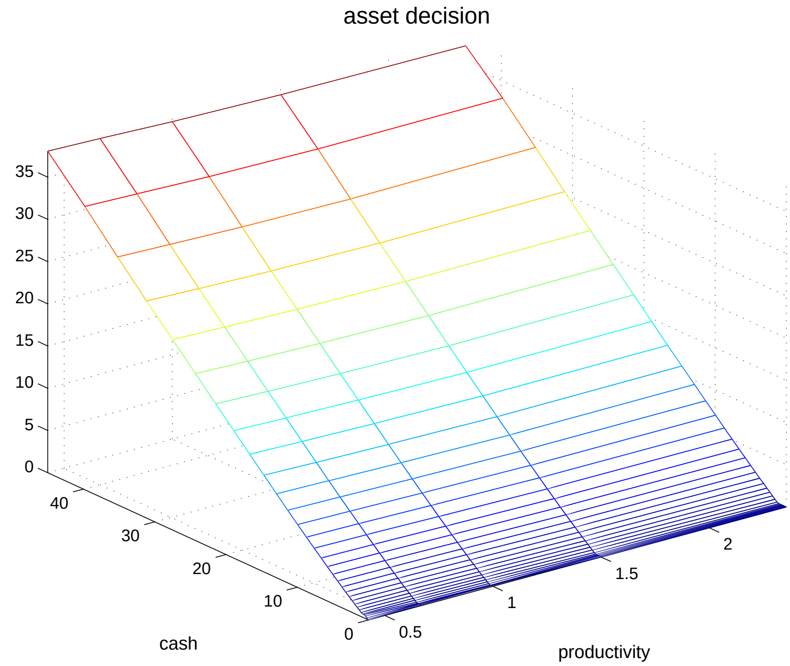


Figure 7: Financial Asset Policy

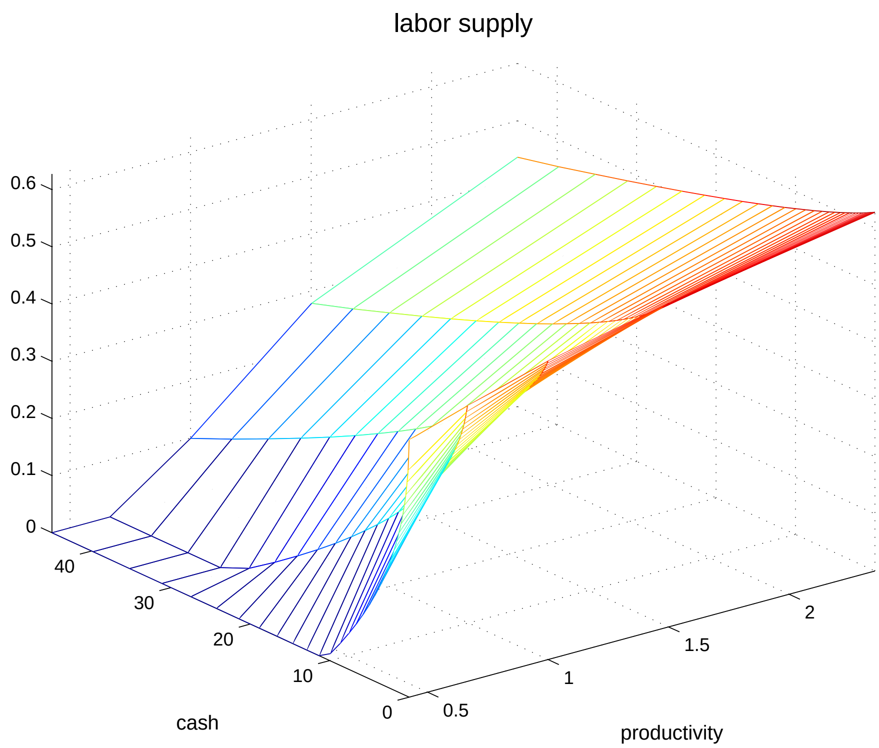


Figure 8: Labor Supply Policy

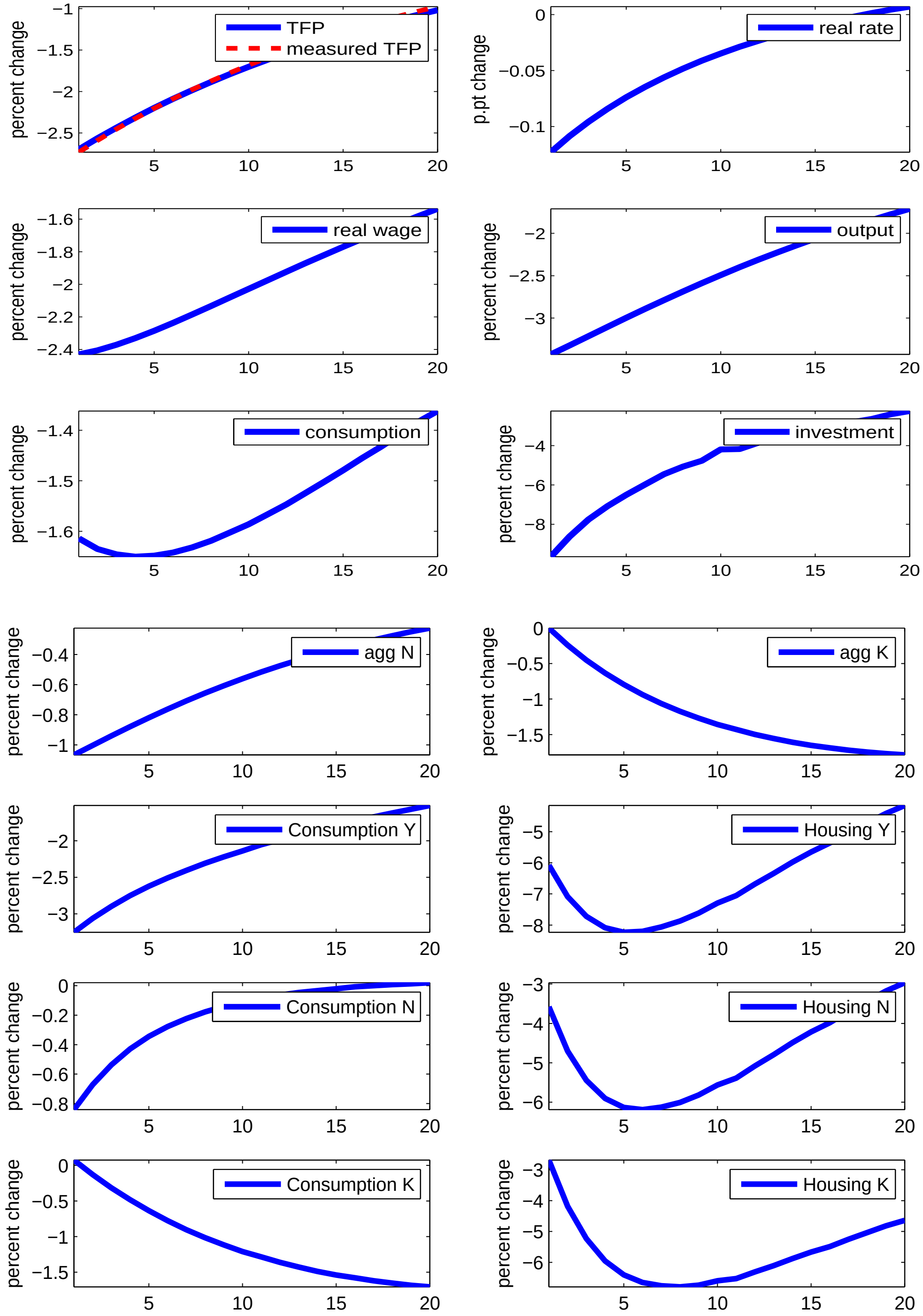


Figure 9: Negative Productivity Shock

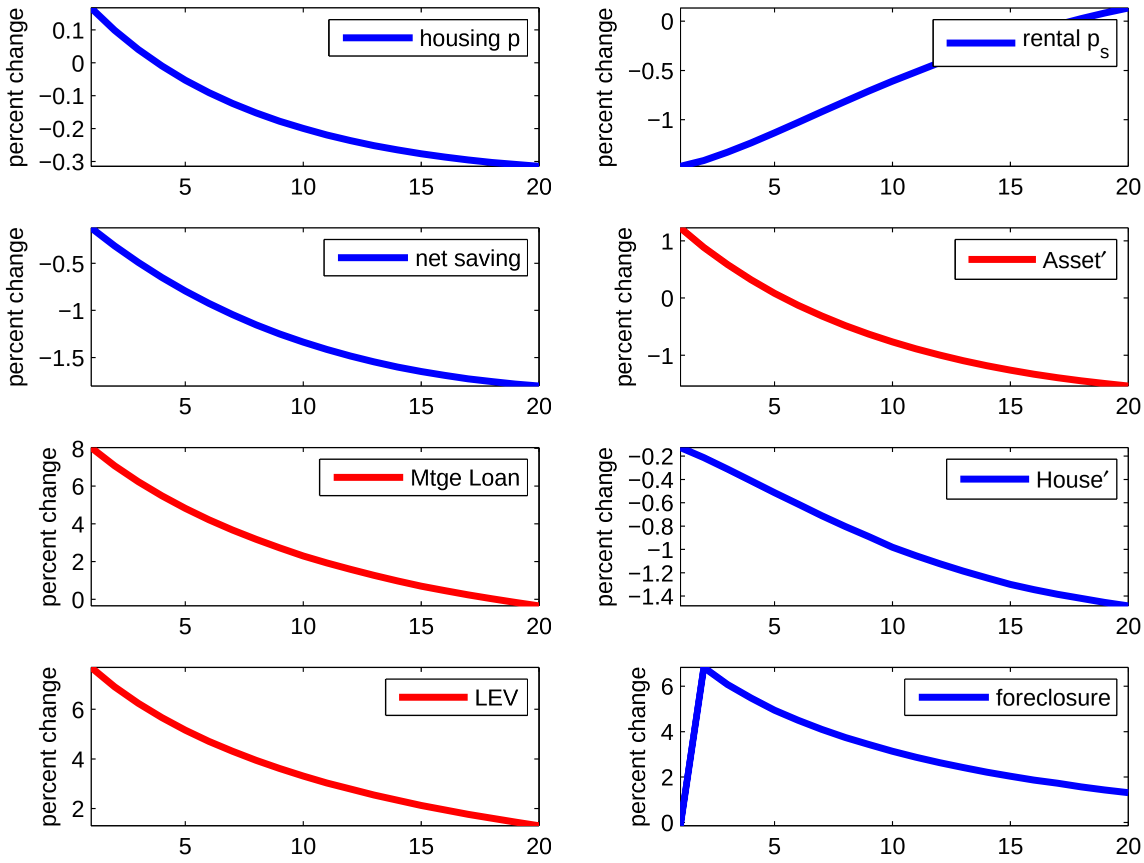


Figure 10: Negative Productivity Shock: Housing Market Variables

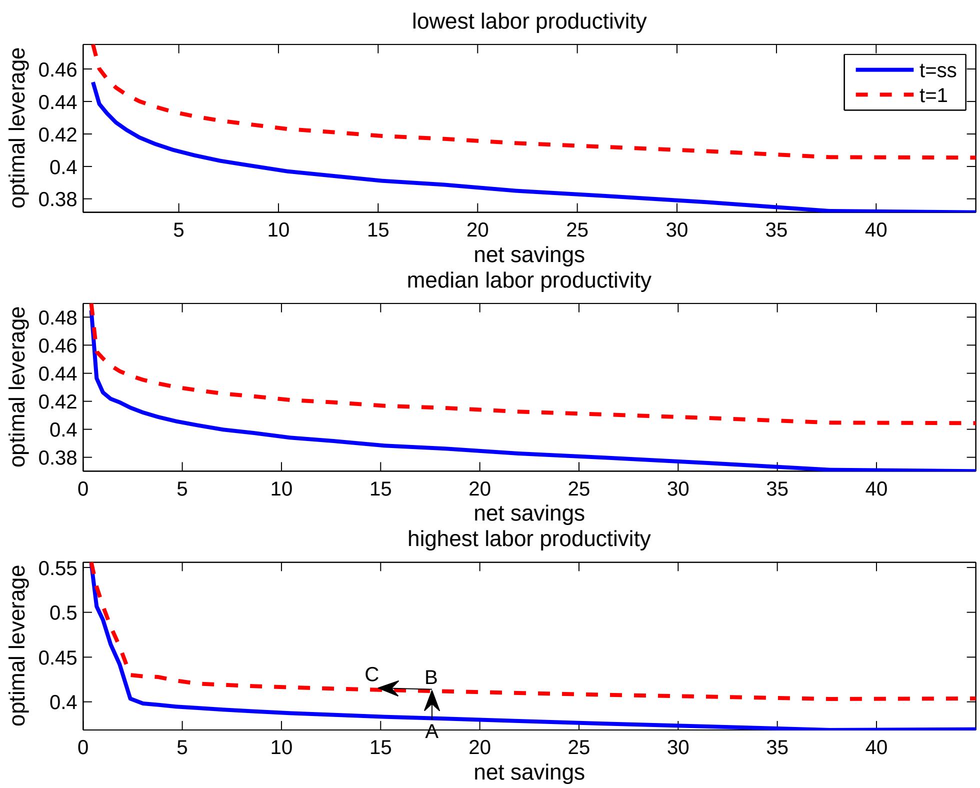


Figure 11: Shifts of Household Leverage Policy in the Productivity Shock

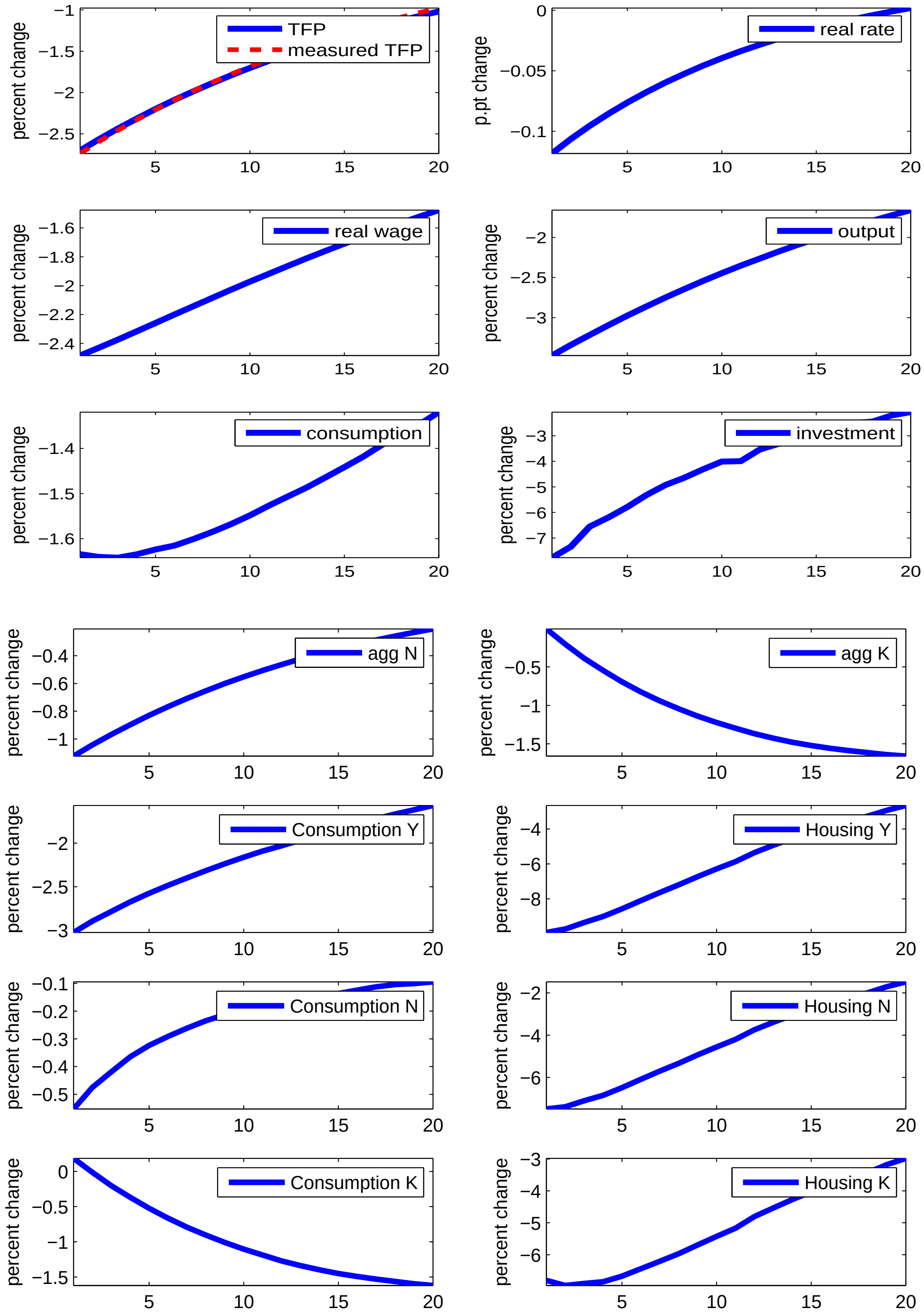


Figure 12: The Great Recession Experiment

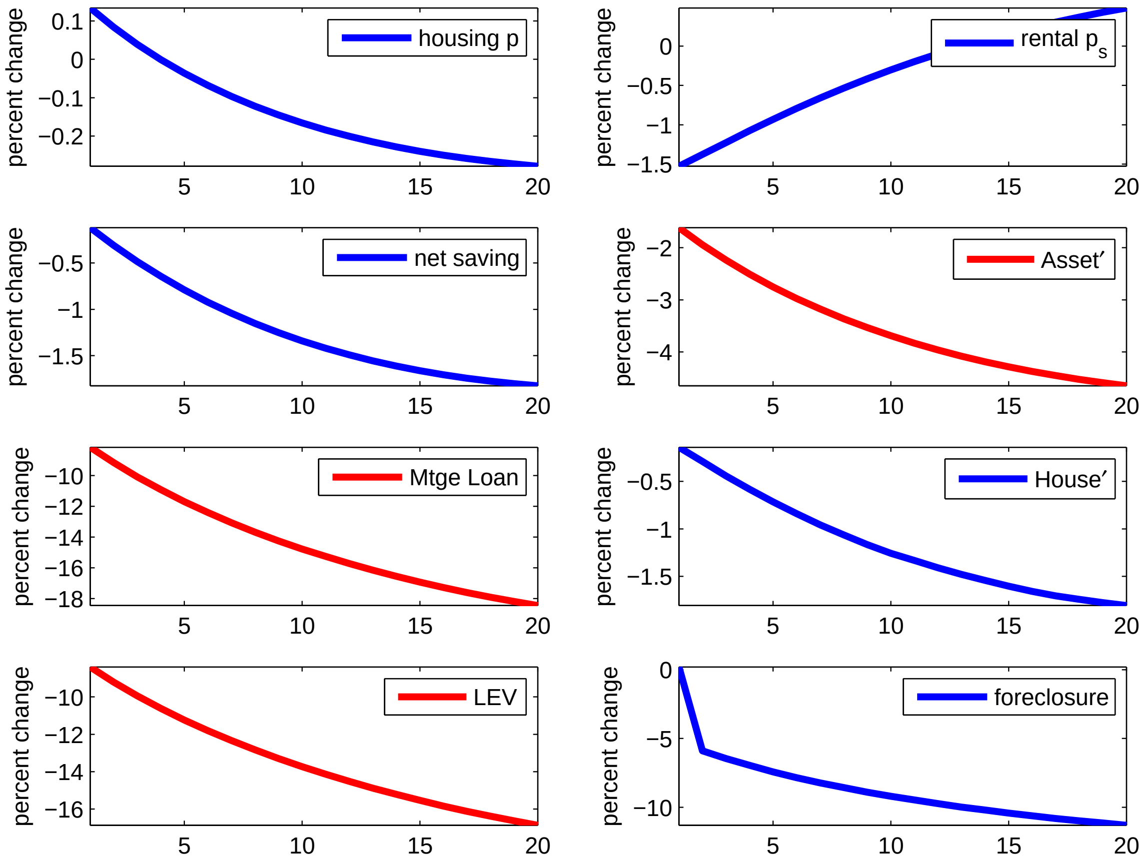


Figure 13: The Great Recession Experiment: Housing Market Variables

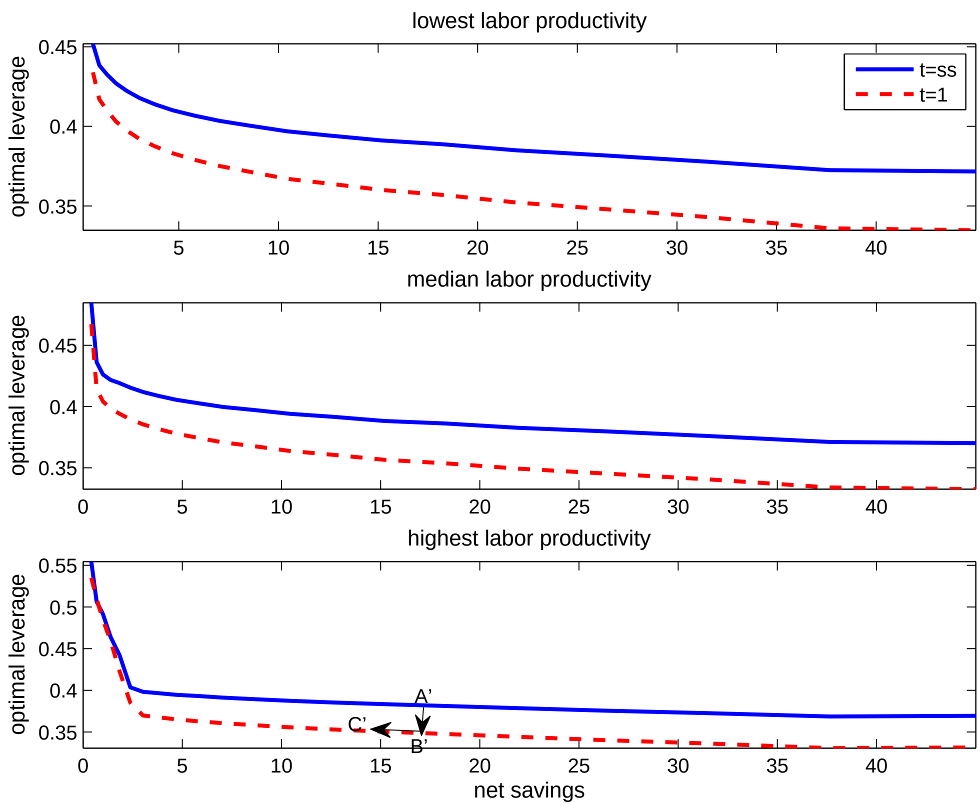


Figure 14: Shifts of Household Leverage Policy in the Great Recession Experiment

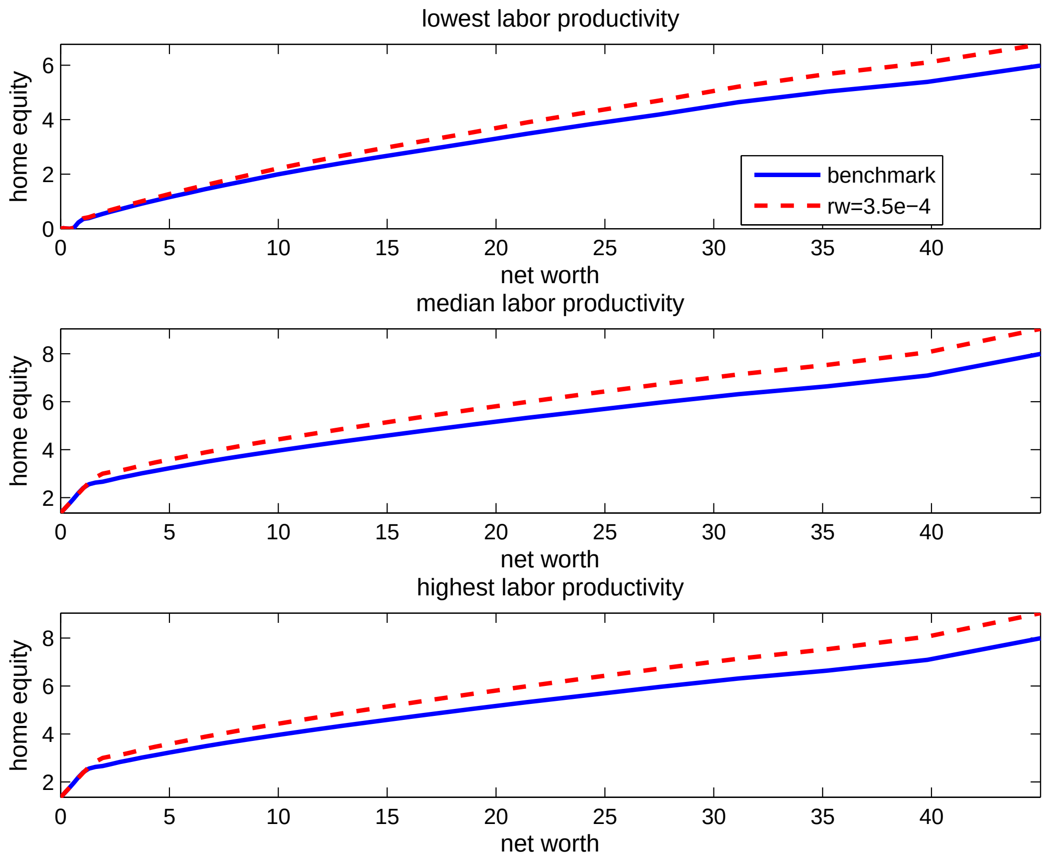


Figure 15: Home Equity with the Tighter Financial Condition ($r_w = 3.5e - 4$)

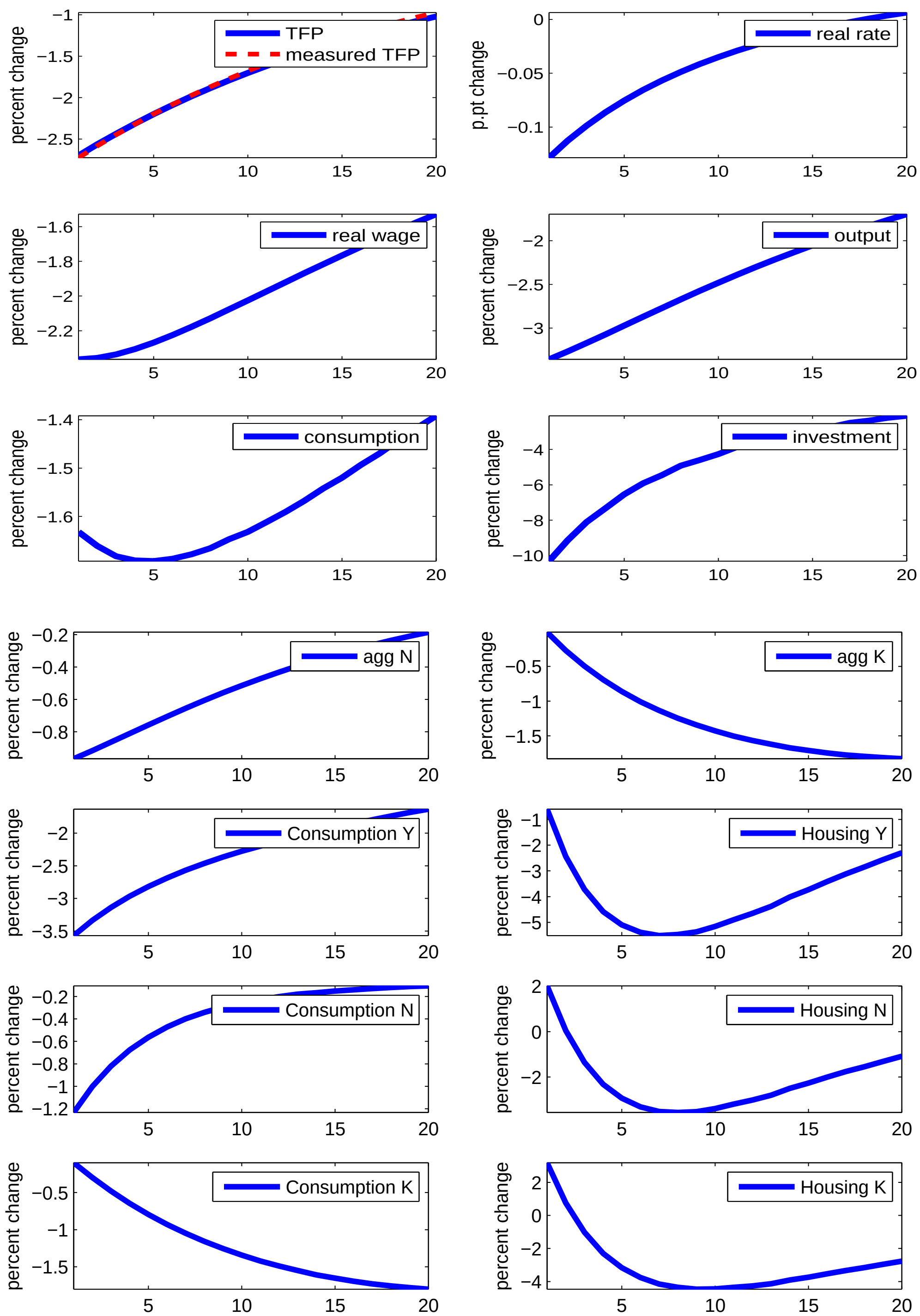


Figure 16: The Foreclosure Boom Experiment

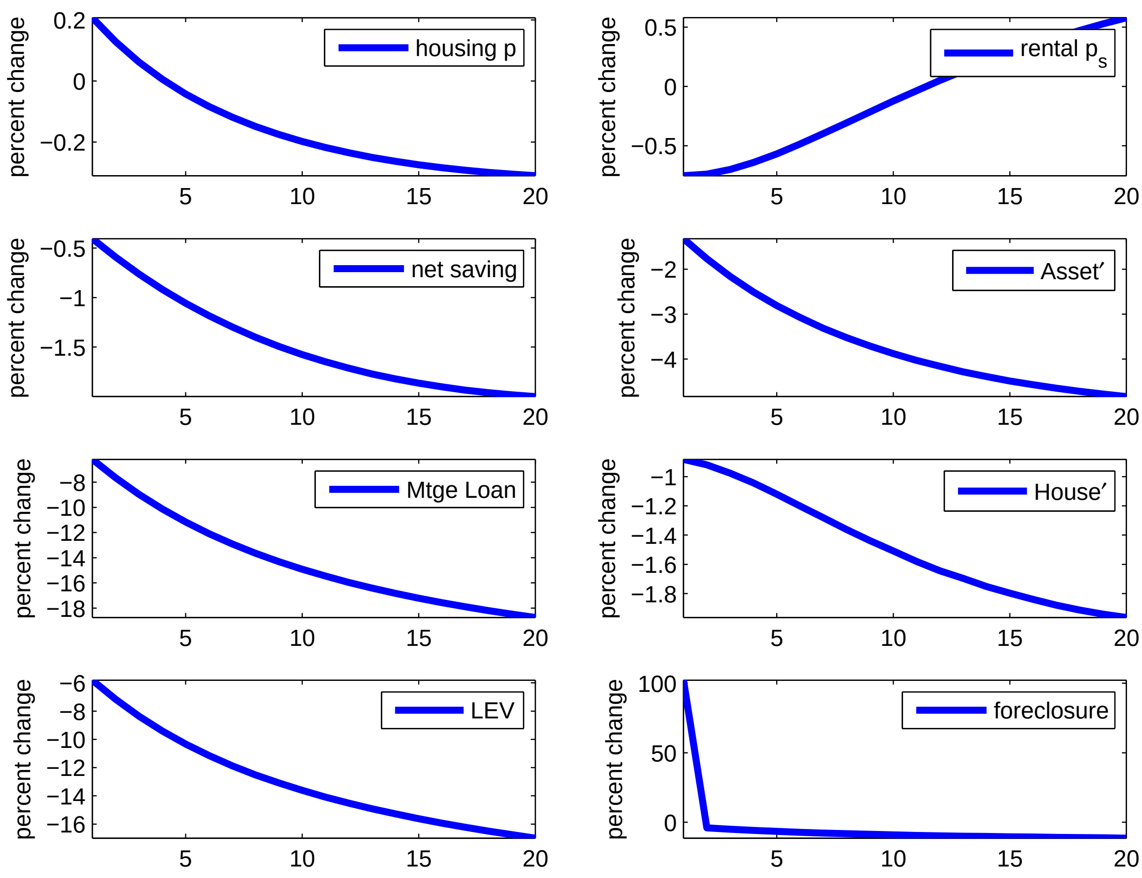


Figure 17: The Foreclosure Boom Experiment: Housing Market Variables

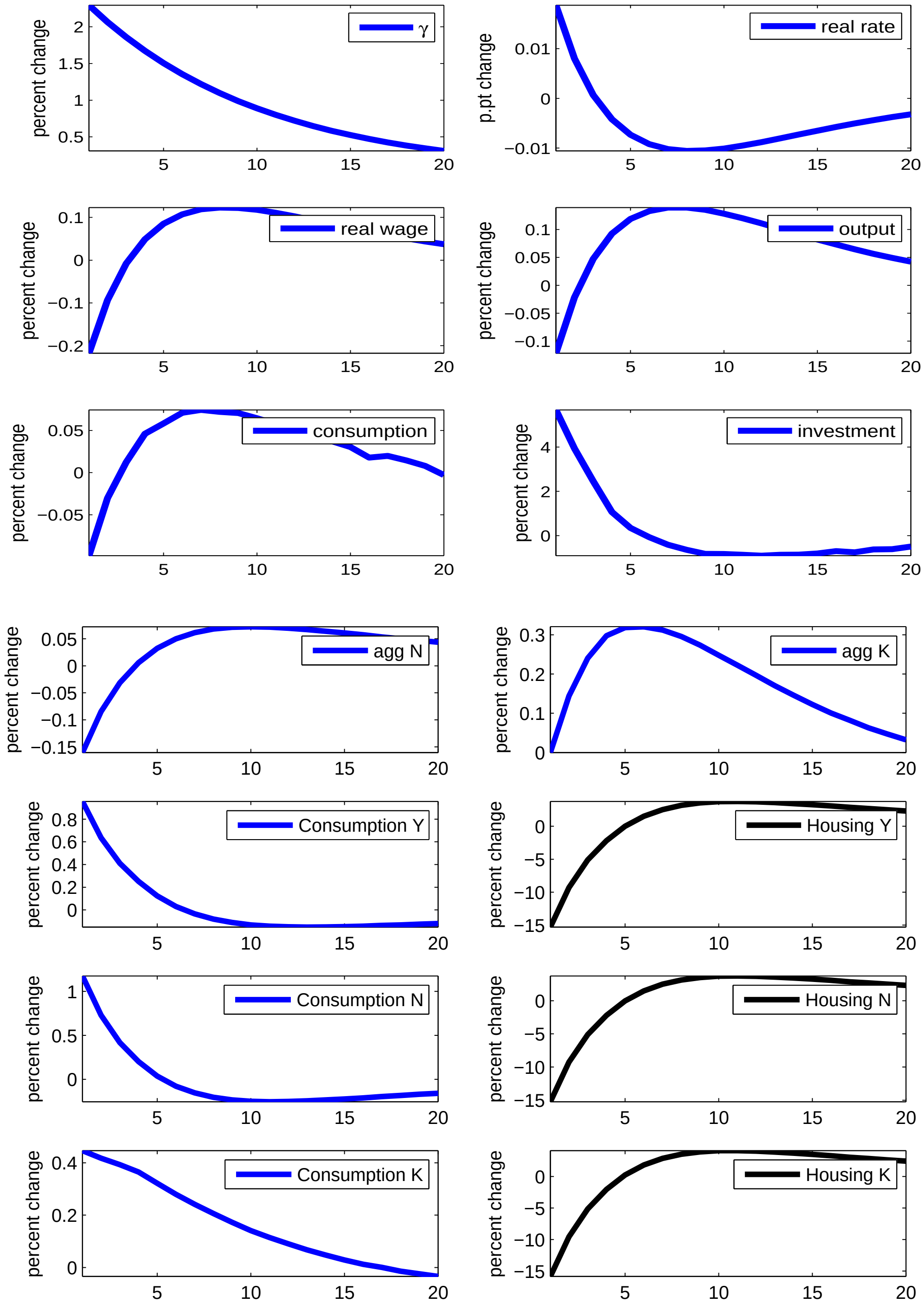


Figure 18: The Depreciation Shock Experiment

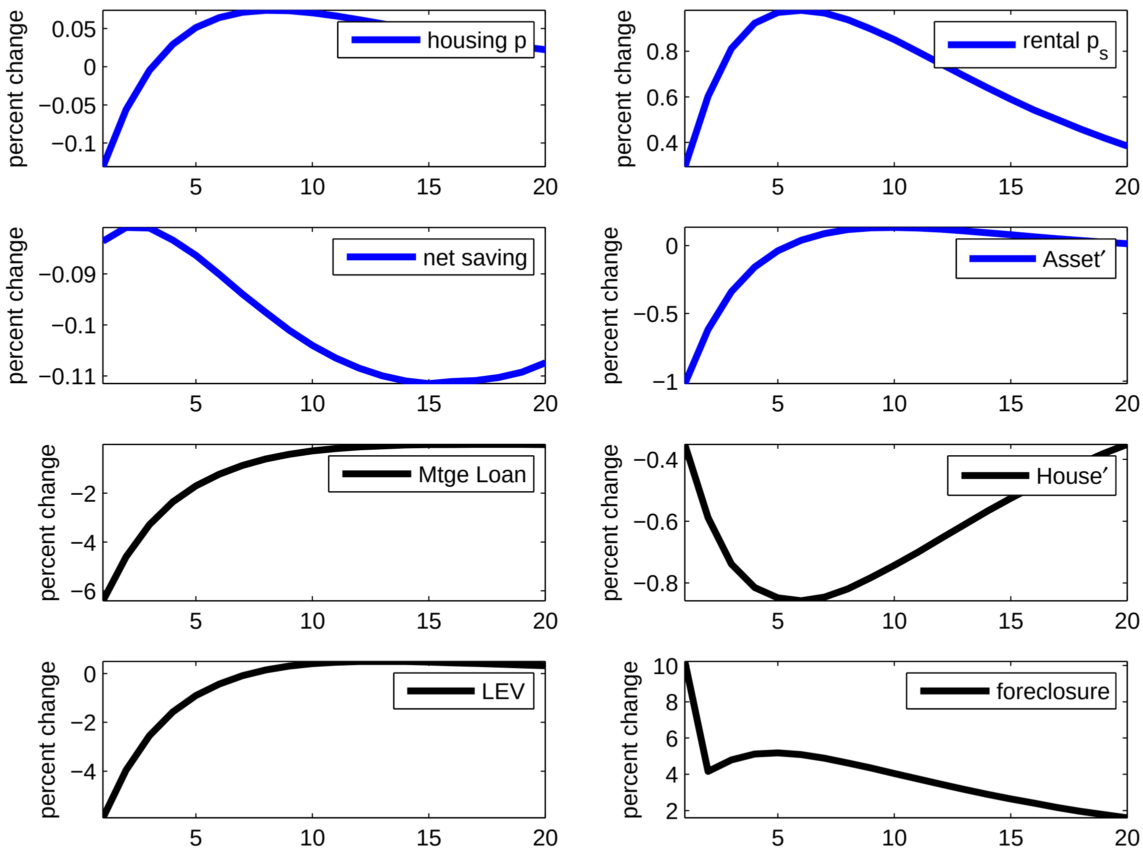


Figure 19: The Depreciation Shock Experiment: Housing Market Variables