Heterogeneous Households, Mortgage Debt and Housing Prices over the Great Recession*

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Abstract

This paper studies the contractions of the U.S. economy over the Great Recession with declines in housing variables. I build a quantitative general equilibrium model with heterogeneous households and two sectors. Households face portfolio problems that involve selecting the stock of housing, mortgage debt and financial asset. The real house prices are endogenous and households have the options to default on mortgage debt. The model matches the housing and non-housing moments in the U.S. data such as the capital-output ratio and the mortgage foreclosure rate. I find that the combination of a negative productivity shock, a mortgage financial shock and a housing over-supply shock on the economy can explain the contractions in the aggregate economy, mortgage debt and part of the declines in house prices. In particular, the productivity shock and housing over-supply can generate large declines in house prices. Furthermore, house prices have significant spillover effects on consumption.

JEL Classification: E21, E32, G11, R21, R23

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1 Introduction

The Great Recession started with the decline of housing market and was later followed by severe contractions in output, consumption, and investment. Although the housing market is closely tied to the Great Recession, the mechanism of the recession with declines in housing have not been fully studied. The objective of this paper is twofold. First, I reproduce the economic recession with declines in housing variables in a dynamic stochastic general equilibrium (DSGE) model. Second, I explore whether the housing variables have any spillover effects over the recession. Two key aspects distinguish my analytic framework from existing housing models over the Great Recession. First, I consider household heterogeneity in both housing and financial assets. Second, house prices are endogenous and mortgage default is allowed.

Specifically, I study a heterogeneous agent general equilibrium model with incomplete markets based on Jeske, Krueger and Mitman (2013) in the tradition of Bewley (1986), Huggett (1993) and Aiyagari (1994). The economy is populated by infinitely lived heterogeneous households with idiosyncratic labor productivity that solve consumption, labor and portfolio problems each period. The portfolio problem involves selecting the stock of housing, mortgage debt and non-housing/financial asset to maximize expected lifetime value from consumption, housing service and leisure. Houses can serve as collateral to take out mortgages, and households are allowed to default on their mortgage debt. Banks issue mortgages and price the default risk and housing value risk into the mortgage interest rates they offer. To allow for variations in real house prices, the model includes two sectors: a consumption good sector and a housing good sector. Both sectors produce using capital and labor as inputs. The relative price of housing goods using consumption good as the *numeraire* is the endogenous real house prices.

The model can be calibrated to match the empirical housing and non-housing moments such as the capital-output ratio, loan-to-value ratio and mortgage foreclosure rate in the U.S. data. With the parameters identified in calibration, the steady state benchmark model is able to generate financial asset to housing asset ratio, housing wealth to GDP ratio, residential investment to GDP ratio and homeownership rate that are close to their correspondences in the U.S. data. The steady state economy resides households that would save risk-free financial assets and borrow through mortgages simultaneously.

In the paper, the housing market is characterized by several features. First, I consider non-recourse state as recourse is highly limited in practice in most U.S. states.[1] Second,

[1] Twelve U.S. states have non-recourse laws before the 2007-2009 financial crisis. Also, Feldstein (2008) reports that resource is highly limited in practice in most U.S. states.

households derive utility from housing services rather than from owning houses. Homeownership itself does not generate utility since households can rent or/and own houses. Houses generate rental incomes and serve as collateral to take out mortgages. Third, empirical evidence suggests that transaction cost in housing purchase and selling are very low. Thus housing transaction costs are not considered.[2] Finally, houses are perfectly divisible.

To understand the declines in housing and non-housing variables, I consider three factors that seemed relevant for the Great Recession: the reduction in aggregate productivity, the over-supply of housing, and the financial tightening in the mortgage market. Aggregate productivity shock is considered because TFP is measured to be about 2.5 percent lower during the recent financial crisis. Housing over-supply is considered as vacant houses for sale increased from about 1.0 million units in 2000Q1 to 2.29 million units in 2008Q2. Moreover, housing inventory remained above 2 million units throughout the Great Recession until 2011Q2. Furthermore, a disruption in the mortgage market is considered as real outstanding mortgage debt falls significantly and the average mortgage origination fees increased to 0.691 percent since 2008Q3. In contrast, average origination fees for mortgage debt was 0.569 percent from 2002Q1 to 2008Q2.

Incorporating these three factors into the model, I find that they generate a severe economic recession, which is accompanied by significant declines in house prices and mortgage debt. Specifically, they explain most of the declines in output and consumption, part of the decrease in house prices and the entire decrease in mortgage debt. I then use the model to assess the marginal contribution of each factor.

The productivity shock plays a major role in accounting for the contractions in aggregate output and consumption. In its absence, output would decrease only 0.3 percent and consumption decline 0.4 percent. An interesting finding is that the reduced productivity can generate significant decreases in house prices because household housing demand falls when their labor and interest incomes are reduced by the lower productivity. The productivity shock also matters for the housing foreclosure rate as households experience reductions in income.

The over-supply of houses is important for the house prices dynamics. In its absence, house prices only decline by one-third as much as its decline in the baseline experiment. Thus, the housing supply shock accounts for about two-thirds of the decline in house prices in the model. Although housing over-supply only have small effects on output, it contributes significantly to the contraction in consumption.

[2] Chambers, Garriga, and Schlagenhauf (2009) document that the transaction cost in the housing market has decreased from 2% of the purchase price to less than 0.5% since 1985. Li (2005) also finds that transaction cost in the U.S. housing market has greatly reduced.

The decline in house prices have a significant spillover effect on consumption as household become poorer with smaller housing wealth.[3] Specifically, when house prices decline 1% in the model experiment, consumption is estimated to decrease 0.17% on average. This spillover effect is about the same amount to its empirical counterpart.

The tighter mortgage market is key for the significant contraction in mortgage debt (and leverage). In its absence, mortgage debt would increase as households become poorer. Under the tighter mortgage market, foreclosure rate would fall eventually as household take smaller leverage and borrow less even if it increases in the first few quarters.

The algorithm I used to compute the equilibrium extends Jeske et al (2013)'s solution methods to accommodate two sectors and endogenous house prices. Allowing for endogenous real house prices in a decentralized model with heterogeneous households and mortgage default add substantial complication in solving the transitional dynamics as households are forward-looking in their housing and financial asset decisions. Specifically, households consider future house prices and interest rates when they make current purchases of houses and financial assets. Thus future prices have impacts over the current resource allocations and aggregates. Therefore, the equilibrium prices in the current period can be affected by household expectations over the future prices.

There are three aspects in my study of the Great Recession that are worth pointing out. First, the three shocks are independent of each other and the marginal contributions of shocks refer to their contributions in accounting sense rather than causal sense. This approach is also applied in Chatterjee and Eyigungor (2015). Second, my analytic framework captures the mechanism for the severity and persistence of the recent recession, but the goal of this paper is mainly methodological. I explore a mechanism that can account for a major part of the contractions in output, consumption, mortgage and part of the declines in house prices, but abstract from other channels and factors.

Third, in the model negative home equity is necessary and sufficient for household to default as in Jeske, Krueger and Mitman (2013). The advantage of this feature is that it provides an explicit default decision rule that delivers insights into the major reason why households default and simplifies the numerical analysis. Other factors, such as income, could also matter for household mortgage default decisions. Gerardi et al (2009) and Buttha et al (2010) find that most defaults involve negative equity, but not all households with negative equity choose to default.[4] These evidence suggests that a second trigger such as negative income shocks might be necessary for foreclosure to take place. Although negative income

[3]Since nonhousing and housing consumption have a fixed proportional relationship in the data, the spillover effect of housing wealth is on both types of consumption.

[4]Moreover, defaults do not necessarily require negative equity in the houses. Low (2014) documented that about 30-50% of mortgage defaulters in the recent recession have positive equity.

shocks do not directly generate default in my model, households that experience reductions in income tend to take higher leverage and thus have higher probability to obtain negative home equity and default. Therefore, negative income shocks can indirectly trigger default in my model. For this reason, households with lower net worth and labor productivity default more frequently than other households in the steady state.

To the best of my knowledge, mine is the first paper that considers heterogeneous households, mortgage default and endogenous house prices simultaneously in a DSGE model to study the mechanism of the recent recession. Also, the paper contributes to advance our understanding on the spillover effects of housing variables. As noted by many observers, the origin, severity and transmission of the Great Recession is closely tied to the low house prices and high defaults, but we lack a mechanism to describe it. This paper serves as the first step to disentangle the interactions between the housing market and the aggregate economy in a DSGE framework with household heterogeneity in housing, mortgage and financial assets. Thus the paper might provide researchers that study topics about the Great Recession involving the housing markets a channel to introduce the Great Recession and a sense of different channels that a more comprehensive theory of the Great Recession may draw upon.

1.1 Related Literature

This paper is first related to business cycle literature with housing production. Leading examples are Benhabib et al (1991), Davis and Heathcote (2005), Iacoviello and Neri (2010), and Fisher (2007). Iacoviello and Neri (2010) develop a DSGE model with housing that estimates and studies the U.S. housing market from 1965Q1 to 2006Q3. They find a nonnegligible spillover effect from the housing market on consumption. This paper go beyond them to study the Great Recession with housing by distinguishing owning and renting, considering mortgage default and household heterogeneity in housing and non-housing wealth.

My paper is closely related to papers that study the housing market with heterogeneous agents such as Gervais (2002), Chatterjee and Eygungor (2015), Corbae and Quintin (2015), Garriga and Schlagenhauf (2009), Sommer and Sullivan (2015), and Jeske et al (2013). Gervais (2002) studies the distortions of housing taxation on the composition of aggregate capital. Jeske et al (2013) builds a heterogeneous agents model with endogenous mortgage default options to study the macroeconomic and distributional impact of subsidies from Government Sponsored Enterprises. They find that eliminating the subsidies leads to a substantial reduction of mortgage origination and increases aggregate welfare. Their seminal work provides a useful framework on housing and mortgage markets with collateralized default and mortgage pricing. My paper differs from theirs in three key respects. First, I depart from their endowment setting to include two sectors that produce consumption goods and housing goods respectively. Second, the real house prices are endogenous in my model with consumption good as the *numeraire*. Third, this paper aims to study the impact of reduced productivity, financial tightening and high housing supply on the aggregate economy and the housing market in order to improve our understanding about the Great Recession. However, Jeske et al (2013) is silent about this aspect.

Iacoviello (2005) and Iacoviello and Pavan (2013) stand between the previous two streams of housing literature. Iacoviello and Pavan (2013) study the procyclicality of housing demand and mortgage debt over the business cycle and find that higher individual income risk and lower down payments can explain part of the reduced volatility of GDP. My model also succeeds in generating declines in output and mortgage debt and is thus complementary to Iacoviello and Pavan (2013). My contribution, relative to theirs, is to introduce endogenous house prices and mortgage default to explore the mechanism of economic recession with declines in housing variables and the spillover effects of housing variables on consumption quantitatively.

The paper is also related to the literature that studies the effects of an increase in the mortgage foreclosure rate on the aggregate economy. A leading example is Forlati and Lambertini (2011). They find that an unanticipated increase in mortgage risk can be amplified in an economy with high-leverage in mortgage debt and cause output and consumption to decline. My study departs from theirs in several dimensions. First, I consider heterogeneous households with incomplete markets rather than two households that differ in terms of discount factor. Thus I can have homeownership rate and the household wealth distribution. Second, they assume that housing serves as the unique asset for borrowers, there is no housing rental, and productions are carried out using labor as the unique input. As a result, they cannot explain the decline of labor, investment and financial asset. Third, they are silent about the spillover effects from the housing market and the real effects of financial shock, productivity shock and housing over-supply over the economy.

The rest of the paper is organized as follows. Section 2 presents the baseline model. Section 3 shows parametrization. Section 4 summarizes the steady state results. Section 5 presents the transitional dynamics of the baseline experiment. Section 6 discusses the contributions of shocks. Section 7 estimates the house prices spillover effect. Section 8 presents robustness test. Section 9 concludes.

2 The Model

2.1 Environment

There is a continuum of households that are indexed by $i \in [0,1]$. Each household is endowed with one unit of time to divide between labor and leisure. Households live infinitely and have idiosyncratic labor productivity ϵ. Households save using two kinds of assets. Firstly, households can hold risk-free financial asset a which earns risk-free interest rate r per unit of assets saved. Secondly, households can purchase perfectly divisible housing asset h. However, houses are risky assets as they are subject to idiosyncratic housing value depreciation shocks.[5] Let δ' denote the housing value depreciation shock tomorrow. Depreciation δ' is an i.i.d. draw across time for every household from the continuously differentiable cumulative distribution function $F(\delta')$, $\delta' \in [\underline{\delta}, 1]$.

There is a competitive housing rental market where households can trade housing services. One unit of housing asset generates one unit of housing service. A house purchased at the beginning of a period can be rented out immediately and thus generate rental income in the same period as the purchase. Short selling of risk-free non-housing assets and houses are prohibited.

Households can use housing assets as collateral to take out mortgages issued by the banks. Let m' denote the size of the mortgage, and p_m denote the mortgage price. A household that enters the next period with (h', m')[6] has the option to default on its mortgage payment after observing the housing price p'. If the household chooses to default, the punishment is losing the ownership of the house to the banks. A defaulting household is not punished in any other form in the financial market. There is no recourse state and no transaction cost in housing purchases and sales. Given these assumptions, a household chooses to default if and only if the housing asset is underwater, i.e., if the housing value is smaller than the mortgage payment. That is,

$$p'(1-\delta')h' < m' \tag{1}$$

Equation (1) is the household default decision rule. It implies that the ex-ante default probability at the origination of the mortgage prior to observing the depreciation only depends on the size of the mortgage m' and housing value $p'h'$.[7] Thus mortgage price p_m is simply a function of (m', h'). It also implies that the cutoff housing depreciation rate at which a

[5] In reality the depreciation might correspond to changes in the neighborhood environment such as the quality of neighbors, convenience stores, public parks, or physical damages/renovations on the houses, etc.

[6] h', m' are household state variables for the next period. The default decision rule here creates an explicit relationship among housing, mortgage and net worth: $x' = (1+r')a' + \max\{0, p'(1-\delta')h' - m'\}$. Moreover, household consumption and saving decisions depend on their net worth directly. Thus I track the net worth rather than h', m' in the household problem.

[7] See Jeske et al (2013) for a more detailed discussion.

household is indifferent between defaulting and repaying is $\delta^* = \max\left\{\underline{\delta}, 1 - \frac{m'}{p'h'}\right\}$.

2.2 Household problem

Let x denote household net worth which is the real value of all assets brought into the period after the housing depreciation shock has materialized. Let ϵ denote the household labor productivity. Households thus have two individual state variables (x, ϵ). Let $\mu(x, \epsilon)$ denote households distribution over individual state variables (x, ϵ). Then aggregate state variables are (z, λ_h, μ), where z and λ_h represent productivity in the consumption and housing sectors. Since the main interest of the paper is the stationary economy and perfect foresight transitions, the dependence of prices on (z, λ_h, μ) are left implicit. In each period, households maximize discounted expected lifetime value from consumption c, leisure $(1-n)$, and housing services s taking real interest rate r, real wage rate w, real rental price p_s and mortgage price function $p_m(\cdot,\cdot)$ as given. That is, households solve the following problem

$$V(x, \epsilon) = \max_{c,s,a',h',m',n} u(c, s, 1-n) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon) \int_{\underline{\delta}}^{1} V(x', \epsilon') dF(\delta') \tag{2}$$

subject to

$$c + p_s s + a' + ph' - m' p_m(m', h') = w\epsilon n + x + p_s h' \tag{3}$$

$$c, s, a', h', m' \geq 0, \quad 0 \leq n < 1 \tag{4}$$

where net worth $x' = (1 + r')a' + \max\{0, p'(1-\delta')h' - m'\}$

Equation (3) is the household budget constraint. The right hand side of equation (3) denotes resources available to the household within the period including labor income and net worth. Since the timing is that houses purchased this period can be rented out immediately, household rental income $p_s h'$ also shows up as part of the household resources within the period. The left hand side of equation (3) is the household allocation of resources among consumption, housing service and asset portfolio which involves selecting the level of financial assets, housing assets and mortgage debt.

Future net worth x' consists of financial asset income and home equity. If future housing value after the realization of housing depreciation is larger than the mortgage debt, home equity is positive and equals $p'(1-\delta')h' - m'$. In this case, households repay the debt. Otherwise, household home equity is zero and net worth $x' = (1 + r')a'$ as households use the default option and have their houses foreclosed on.[8]

[8] In reality, it is possible that home equity becomes negative and households do not trigger default for various reasons. For example, there are penalties on household credit report if they default. Homeownership itself might be valuable to the households and it involves losses of additional resources to move to new places. In the model, existing assumptions eliminate

2.3 The Banking Sector

Assume that banks are perfectly competitive and have the technology to convert risk-free assets into capital without any cost. At the beginning of each period, banks take deposits of financial asset from households, lend capital to the housing production sector and issue mortgages. Following Jeske et al (2013), I assume that issuing mortgage is costly so that banks have to lose an additional r_w units of real resources per unit of mortgage issued. r_w characterizes the screening, monitoring, administrative as well as the maintenance costs associated with each unit of mortgage. Thus the effective cost of issuing a unit of mortgage equals $r + r_w$ and banks discount the expected payments received next period at $\frac{1}{1+r+r_w}$.[9] When households default, banks seize the after depreciation housing value. However, the bank foreclosure process is costly and only recovers a fraction $\theta \in [0, 1]$ of the collateral value.

Banks take into account that households might default on the mortgage payments next period. Therefore, mortgage price is such that each mortgage contract compensates for the expected loss in the case of default.

$$m' p_m(m', h') = \frac{1}{1 + r + r_w} \left\{ m' F(\delta^*) + \theta p' h' \int_{\delta^*}^{1} (1 - \delta') dF(\delta') \right\} \tag{5}$$

where $\delta^* = \max \left\{ \underline{\delta}, 1 - \frac{m'}{p'h'} \right\}$ is the cutoff housing depreciation rate at which a household is indifferent between defaulting and repaying.

In equation (5), $m' p_m(m', h')$ is the actual units of consumption that a household obtains when he takes out a mortgage of size m' and buys a house of size h'. The right hand side is the expected discounted revenue that banks receive next period from (m', h'). With probability $F(\delta^*)$ household receives a housing value depreciation shock δ' that is lower than the threshold depreciation δ^* so that repay the mortgage is optimal. With probability $1 - F(\delta^*)$ households default and banks liquidize the house after a costly foreclosure process which only recovers θ fraction of the after depreciation housing value.

2.4 Representative Production Sectors

There are two representative production sectors in the economy, a consumption good sector and a housing good sector. Assume that capital and labor are perfectly mobile. The consumption good sector produces consumption goods using capital and labor according to

these possibilities and default is chosen if and only if the housing asset is underwater. Thus, home equity is always nonnegative in the model.

[9] As in Jeske et al (2013), r_w is paid when the mortgage is repaid. When a household defaults on a mortgage payment, it also defaults on the mortgage issuance cost.

the production technology $Y_c = zK_c^{\alpha}N_c^{1-\alpha}$. Thus, the representative consumption good firm solves the following problem

$$\max_{K_c, N_c \geq 0} \left\{ zK_c^{\alpha}N_c^{(1-\alpha)} - (r+\delta_k)K_c - wN_c \right\} \tag{6}$$

The housing sector produces new homes using capital and labor according to the production technology $I_h = \lambda_h K_h^{\nu} N_h^{1-\nu}$. Let δ_k denote capital depreciation and p be the real housing price with consumption good as the *numeraire*. The representative firm in the housing sector solves

$$\max_{K_h, N_h \geq 0} \left\{ p\lambda_h K_h^{\nu} N_h^{1-\nu} - (r+\delta_k)K_h - wN_h \right\} \tag{7}$$

Thus the aggregate output in the economy is $Y = Y_c + pI_h$. The above two static maximization problems imply that profits are maximized by choosing K_c, N_c, K_h N_h such that

$$r = \alpha z K_c^{\alpha-1} N_c^{1-\alpha} - \delta_k \tag{8}$$

$$w = (1-\alpha) z K_c^{\alpha} N_c^{-\alpha} \tag{9}$$

$$p = \frac{1}{\nu\lambda_h}\left(\frac{\nu}{1-\nu}\right)^{1-\nu} (r+\delta_k)^{\nu}\, w^{1-\nu} \tag{10}$$

2.5 General Equilibrium

A recursive competitive equilibrium consists of a set of functions

$$(p, p_s, p_m, r, w, V, c, s, n, a', h', m', N_c, N_h, K_c, K_h, \mu) \tag{11}$$

that satisfies the following conditions.

(i) Given prices p, p_s, p_m, r and w, the value function V solves (2) and c, s, n, a', m', h' are the associated policy functions

(ii) Given prices, policies K_c, N_c solves the consumption good production problem and N_h, K_h solves the housing production problem

(iii) Given $p_m(\cdot,\cdot)$, financial intermediaries break even for all (m', h')

(iv) The consumption good market clears

$$\int c d\mu + I_b = Y_c \tag{12}$$

where $I_b = I + (r_w + r_{-1} - r)\int p_m(m,h) m d\mu$ is the broad investment and $I = \bar{K}' - (1-\delta_k)\bar{K}$ is the investment in the aggregate capital stock. r_{-1} is the real interest rate in the previous period and $\int p_m(m,h)md\mu$ is the aggregate mortgage loan that is borrowed by households in the previous period. $\bar{K}$ is the aggregate capital stock.

(v) The housing rental market clears

$$\int s d\mu = \int h' d\mu \tag{13}$$

(vi) The labor market clears

$$N_c + N_h = \int (\epsilon n)\, d\mu \tag{14}$$

(vii) The asset market clears

$$\int a' d\mu = \int p_m(m', h')m' d\mu + \bar{K}' \tag{15}$$

(viii) The capital market clears

$$K_c + K_h = \bar{K} \tag{16}$$

(ix) The housing market clears

$$\int h' d\mu = I_h + H \tag{17}$$

where I_h is the total newly built houses this period and $H = \int \int_{\underline{\delta}}^{\delta^*} h(1-\delta)dF(\delta)d\mu + \int \theta \int_{\delta^*}^{1} h(1-\delta)dF(\delta)d\mu$ is the effective aggregate housing stock after depreciation and the foreclosure process.

(x) The evolution of household distribution over individual variables, $\mu(x,\epsilon)$, is stationary (i.e., given the prices implied by this distribution, individuals' actions reproduce the same measure μ in the following period.)

3 Parameterization

One period in the model is a quarter. The momentary utility function for household is

$$u(c, s, 1-n) = \frac{\left(c^{\tau_1} s^{\tau-\tau_1} (1-n)^{1-\tau}\right)^{1-\sigma} - 1}{1-\sigma} \tag{18}$$

Parameter τ is the share for total consumption which is the sum of consumption on consumption good and housing service. Parameter τ_1 denotes the share for consumption goods.

Suppose the idiosyncratic labor productivity ϵ follows a log AR(1) process

$$log\epsilon_{t+1} = \rho_\epsilon log\epsilon_t + (1-\rho_\epsilon^2)^{0.5}\eta_{\epsilon,t}, \quad \eta_\epsilon \sim N(0,\sigma_\eta^2) \tag{19}$$

I follow Jeske et al (2013) to set the persistence of labor productivity $\rho_\epsilon = 0.98$ and the standard deviation $\sigma_\eta = 0.3$, which are in line with the empirical literature on labor productivity and a vast literature on the nature and specification of the household income process. In Pennington-Cross (2004), the estimates of the average default loss is 22% with national data. I let $\theta = 0.78$ to be consistent with Pennington-Cross (2004). I follow Jeske et al (2013) to set the CRRA parameter $\sigma = 3.9$. To sum up, Table 1 lists the parameters that are adopted directly from the data counterparts.

The rest of the parameters are calibrated together to match eight empirical moments at the same time. To match the empirical housing foreclosure rate, the housing depreciation process $F(\delta)$ is assumed to be a generalized Pareto distribution with probability density function

$$f(\delta) = \frac{1}{\sigma_\delta}\left(1 + \frac{\gamma(\delta - \underline{\delta})}{\sigma_\delta}\right)^{(-\frac{1}{\gamma}-1)} \tag{20}$$

I follow Jeske et al (2013) to adopt the generalized Pareto distribution because the right tail of a log-normal distribution is too thin to generate sufficient foreclosure rate within the context of this model under realistic mean and standard deviation. I calibrate the three parameters γ, $\underline{\delta}$ and σ_δ by targeting three moments in the data: the mortgage foreclosure rate, mean depreciation of residential fixed assets and the standard deviation of housing value. According to the National Delinquency Survey from Mortgage Banker Association (MBA (2006)), the average foreclosure of all mortgage loans that starts in a quarter is about 0.4% from 2002Q1 to 2006Q4. Quarterly foreclosure starts are targeted because there is no delay in the model foreclosure process while in the real world there is often a long delay, which might make the foreclosure rate cumulatively high in the data.[10] The mean depreciation for residential housing is calculated as the consumption of fixed capital in the housing sector divided by the total capital stock of residential housing. The data on the consumption of fixed capital in the housing sector is taken from Table 7.4.5 of National Income and Product Account (NIPA), and the capital stock of residential housing is taken from Fixed Asset Table 1.1.[11] My estimation of the quarterly mean depreciation for residential housing is 1.4%.[12]

[10] According to the Loan Processing Services' Mortgage Monitor report, loans in foreclosure had been delinquent for an average of 255 days. In February 2014 that same average became 956 days.

[11] Table 7.4.5 published by BEA June 25, 2010

[12] This estimation stands in line with Macro-Housing literature such as Jeske et al (2013) and Iacoviello and Pavan (2013).

	Interpretation	Value	Source
δ_k	capital depreciation	0.025	U.S. data
ρ_ϵ	productivity persistence	0.98	Jeske et al (2013)
σ_ϵ	productivity variance	0.30	Jeske et al (2013)
ν	capital's share in housing	0.12	GDP-by-Industry
θ	foreclosure technology	0.78	Pennington and Cross (2004)
σ	CRRA parameter	3.9	Jeske et al (2013)

Table 1: Parameters adopted from the data counterparts

	Target Moment	Model	Target	Data Source
β	risk-free rate	1.25	1.25	U.S. data
τ	Average labor hours	0.33	0.33	U.S. data
τ_1	Consumption's share	0.86	0.86	NIPA
r_w	Aggregate Leverage	0.4	0.4	Flow of Funds
α	Total Capital to Output ratio	2.7	2.5	NIPA
γ	Foreclosure rate	0.39%	0.4%	MBA(2006)
σ_δ	House value volatility	0.07	0.06	OFHEO HPI data
$\underline{\delta}$	Average housing depreciation	1.6%	1.4%	NIPA

Table 2: Calibrated parameters and target data moments

The standard deviation of the housing value is obtained by utilizing the state volatility parameter from the Federal Housing Finance Agency (FHFA or OFHEO). FHFA estimates within-state and within-region annual house price volatility based on Calhoun (1996), which considers house prices to be consisted of a market component and an idiosyncratic component. Following the method in Calhoun (1996), the quarterly volatility estimates for the eight census region is within the broad range of 4-6% from 2000Q1 to 2006Q2. The upper bound 0.06 is used to account for the fact that nationwide house price volatility is slightly higher than the within region volatility.

Moreover, with parameter $\tau = 0.45$, households in the model on average work one-third of their time. Non-housing consumption share $\tau_1 = 0.385$ is chosen so that the share of housing in total consumption expenditure is 14.4%, which is measured using annual data from 1969 to 2001 (NIPA Table 2.4.5). The loan-to-value ratio is about 0.4 from 1969Q1 to 2002Q4 meaning that the mortgage debt is about 0.4 times as large as the housing wealth during that period. The mortgage administration cost $r_w = 0.0001$ is adopted so that the aggregate leverage ratio in the steady state is very close to this target. The time discount factor $\beta = 0.98$ is chosen to hit a quarterly real interest rate of 1.25% in the steady state.

On the production side, parameter $(1 - \nu) = 0.88$ to match labor's share in the construction sector. The average labor's share in the construction sector from 1987 to 2002 is measured to be 0.88 using the method and data source provided in Davis and Heathcote (2005).[13] I choose the capital's share in the consumption good sector $\alpha = 0.3$ such that the aggregate capital to aggregate output ratio $\frac{K}{Y}$ is 2.7 in the steady state and the consumption capital to consumption output ratio $\frac{K_c}{Y_c}$ is about 2.0, which are consistent with the U.S. data. I choose the capital depreciation rate $\delta_k = 0.025$ to be consistent with King and Rebelo (2000). The productivity z in the consumption good sector follows a log AR(1) process

$$log(z_{t+1}) = \rho_z log(z_t) + \zeta_t, \quad \zeta_t \sim N(0, \sigma_\zeta^2) \tag{21}$$

where $\rho_z = 0.95$ as in Bloom et al (2011) and $\sigma_\zeta = 0.0072$ as in King and Rebelo (2000). In the data, construction sector productivity does not show growth since 1964 and it is about half as much as the TFP in all non-farm industries around the year 2000. Thus, I assume λ_h to be a constant parameter and equals to $\frac{1}{2}$.

4 Steady State

In this section, I illustrate the steady state properties of the model. Figure 1 plots the mortgage price function $p_m(m, h)$ provided by the banking sector as described in equation (5). As shown in the figure, the mortgage price is higher when a larger house h is pledged as collateral, holding mortgage m constant. Given the housing asset h, mortgage price decreases as mortgage debt m increases.

Actually, mortgage price is simply determined by household leverage ratio (or loan-to-value ratio). Let $\iota \equiv \frac{m}{ph}$ denote the leverage ratio, then equation (5) can be rewritten as[14]

$$p_m(\iota) = \frac{1}{1 + r + r_w} \left\{ F(1 - \iota) + \frac{\theta}{\iota} \int_{1-\iota}^{1} (1 - \delta) dF(\delta) \right\} \tag{22}$$

By the definition, selecting the optimal mortgage debt is equivalent to choosing the optimal leverage ratio, holding houses h and housing price p constant. Taking derivative with

[13] I do not include land as a production factor in the housing sector. The capital output ratio in the construction sector is estimated to be 0.16. Thus capital here is more appropriate to be interpreted as the combination of capital and land since physical capital itself is almost negligible in the construction sector.

[14] Proposition 2 in Jeske et al (2013) implies that it is never optimal for households to choose leverage $\iota > 1 - \underline{\delta}$ in equilibrium. Thus the threshold depreciation $\delta^* = 1 - \iota$ without loss of generality.

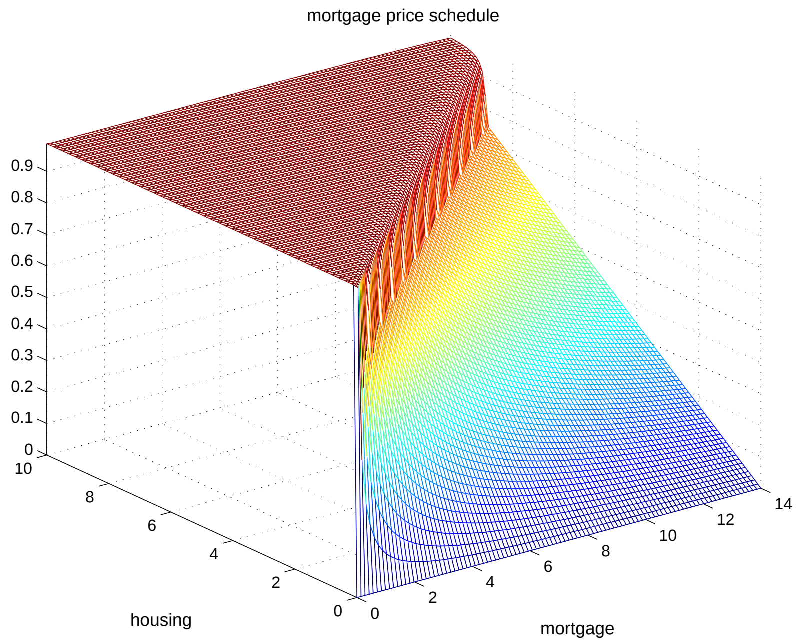

Figure 1: Mortgage Price Schedule $p_m(m', h')$

respect to ι, one can find that $p_m'(\iota) < 0$. Thus the mortgage price is monotonically decreasing in leverage. Intuitively, since the default probability is equal to $1 - F(1 - \iota)$, a larger leverage implies a higher probability of default as the threshold depreciation rate, which is equal to $1 - \iota$, becomes lower. Notice that many factors, such as wealth and income, can affect household leverage decisions and hence can affect the probability of default.

Holding net worth constant, the value function is monotonically increasing in labor productivity. Given labor productivity, the value function is increasing in net worth. Let $g' \equiv a' + (p - p_s)h' - m'p_m(h', m')$, then g' is the net saving from households.[15] By solving a consumption-leisure-savings problem illustrated in the appendix, I find that the net saving policy is also increasing in net worth and labor productivity.[16]

The housing decision is increasing in net worth and labor productivity.[17] Larger net worth and labor productivity means more resources are available to households to allocate between different assets. Under the model framework, households find it optimal to buy a larger house if they have larger net worth. Although households can obtain housing service from renting, households demand risky housing asset because they yield higher expected return than the financial assets. Specifically, the expected return of housing investment comes from two sources: the implicit rental income and the potential appreciation in home value. Since

[15]With this definition, the household problem can be transformed into a consumption-leisure-savings problem in appendix.
[16]See Figure 7 in the appendix.
[17]See Figure 8 in the appendix.

house prices are constant in the steady state, the unique source of housing return is the rental income.[18]

Household leverage policy is decreasing in net worth and/or labor productivity. Leverage is higher for households with little wealth. Leverage then drops quickly as net worth increases until it reaches around 40%. After that, leverage declines mildly because households start to increase their holdings of risk-free non-housing assets.[19] Given the leverage policy, it is households with low net worth and productivity that get more expensive mortgage borrowing.

Households save more risk-free financial assets as net worth increases, but decrease holdings of financial assets when their labor productivity is larger. The reason is that households with little wealth or higher productivity tend to finance their current and future consumption more from labor income. In contrast, high wealth and low productivity household tend to finance current and future consumption more from capital income. Thus high wealth and low productivity households have smaller shares of safe assets in their portfolio.

In the steady state, households save using houses and low-interest financial assets, and borrow through mortgages at the same time. The reason is that households want to take advantage of the higher expected return from owning houses but also try to insure themselves against the adverse idiosyncratic value depreciation shocks. Since the real housing prices are constant in the steady state, uncertainty in housing returns comes from the idiosyncratic depreciation shocks. When households are hit by large depreciation shocks that put their houses underwater, they default and their net worth only depends on how much financial asset they own, $(1+r)a$. To smooth consumption, they find it optimal to hold risk-free financial asset.[20]

Households can benefit from borrowing. When households choose larger leverage/mortgage debt given house h and net worth x, the housing down payment is smaller so that households spend less in purchasing the houses. Thus they can keep a larger amount of their savings as financial assets. Since the financial assets are not seized by the banks when households default, accumulating financial assets enables them to reduce fluctuations in consumption. Thus the benefit of borrowing through mortgages is the increase in household value from increasing and smoothing consumption with risk-free financial assets. On the other hand, the cost of borrowing is the reduction in household value due to larger default risks and net interest payments to the banks.

Both the cost and benefit of borrowing are increasing in leverage. However, the cost of

[18]Given the expected housing depreciation is 1.6%, p equals 3.12 and p_s equals 0.087 in the steady state, the expected housing return is obviously higher than the risk-free interest rate which is equal to 1.25%.

[19]See Figure 9 in the appendix.

[20]Mian and Sufi (2011) has documented that borrowed funds based on home equity are used by households for increasing consumption.

Variable	Interpretation	SS Value	U.S. Data
percent of hhs with $h' > 0$		99.3%	
percent of hhs with $h' > s$	homeownership	50.1%	64%
Wealth Gini	wealth inequality	0.40	0.78
$pH/(4 \times GDP)$	housing wealth	1.0	1.1
Non-housing asset	non-housing asset share	69%	70%
$p \times I_h/GDP$	housing investment share	5.8%	6%

Table 3: Steady State Numerical Results

borrowing rises faster than the benefit given the format of the mortgage price schedule and the concavity of household value function. Thus there exists a unique optimal leverage policy for each given house h and housing prices p. The optimal leverage is such that it equalizes the benefit and cost of borrowing. Therefore, in equilibrium households would borrow through residential mortgages and save using financial and housing assets at the same time.

In the steady state, the model reproduces a housing foreclosure rate of 0.39% which is consistent with the data. Specifically, households that have their houses foreclosed on are mostly those with little net worth and low productivity, because they are the high leverage takers. Since leverage determines default probability and leverage is decreasing in income and net worth, income and/or net worth can indirectly affect default. Specifically, household with lower income and/or net worth is more likely to default in the economy.

The model generally reproduces the U.S. wealth distribution. Wealth in the model is defined as household net worth. Diaz-Gimenez et al (1997) reported that the Gini coefficient of wealth is 0.78 in the 1992 SCF. The wealth Gini coefficient in the steady state of this model is 0.40, which is close to the Gini in relevant literature: Jeske et al (2013) obtains a Gini coefficient of 0.46 in their steady state with a much smaller discount factor. Iacoviello and Pavan (2013) obtains a Gini coefficient equals to 0.53.

In the steady state, housing wealth constitutes 31% of total household wealth, which is consistent with 30% in the data from 1969Q1-2006Q4. Moreover, housing wealth is as large as real GDP in the benchmark economy, close to 1.1 times in the data from 1969Q1 to 2006Q4. In the steady state, 99.3% of households own strictly positive housing assets and 50.1% of households own larger houses than the amount of housing services they actually consume. Since housing is perfectly divisible in the model, I follow Jeske et al (2013) to use the "percent of households with $h' \geq s$" as the proxy to the homeownership rate. Under the assumption that the housing asset is perfectly divisible, this definition is the best proxy to the owner-occupation homeownership in the data. However, the homeownership does not correspond to the traditional concept of owner-occupation. In this paper homeownership means that the households' holdings of housing assets can fully satisfy their own demand for housing services. This definition of homeownership is consistent with Henderson and Ionnides (1983). The homeownership rate in U.S. data is 64% on average from 1965 to 2001. Jeske et al (2013) have a steady state homeownership rate equal to 40%. The homeownership in this paper thus stands between its counterparts in the data and its closest counterparts in the literature. Moreover, households with higher net worth and lower labor productivity are more likely to become homeowners in the economy.

5 The Baseline Experiments

As noted in the introduction, I consider three unexpected shocks to the economy. The shocks include (i) a decrease in productivity z; (ii) an over-supply of houses; (iii) a financial tightening in the mortgage market.

I model the productivity shock as an 2.7% unexpected decline in consumption productivity. The size of decline in consumption productivity is selected such that the measured total factor productivity (TFP) in the aggregate economy decreases about 2.5% initially to match the TFP decline in the data during the Great Recession. After the initial drop, consumption productivity recovers gradually over time according to equation (21).

As real house prices experienced persistent growth, housing inventory increased greatly before the financial crisis as real estate developers built more houses for the high prices. Between 2005 to 2007, the overall housing stock in U.S. increased by 5 million units, but the number of owner or renter occupied houses only increased by 2.9 million units.[21] According to the U.S. Bureau of the Census, the average vacant housing unit for sale is about 2.13 million units from 2006Q3 to 2009Q4. That is about 3.0% of the stock of owner-occupied housing units in 2005.[22] To characterize the housing over-supply in the model, housing supply increase 3% unexpectedly for four quarters in the economy. After that, housing supply declines gradually to the steady state.[23]

The disruption in mortgage financial market is introduced as an increase in mortgage issuance cost r_w.[24] The economic booms and busts before and after 2007 are closely related

[21] Data source: U.S. Bureau of the Census.

[22] According to the U.S. Census Bureau, there is 74 million owner occupied housing units in 2005.

[23] The housing output y_h increase to be consistent with the 3 percent increase in housing supply. Given the housing stock in the steady state, we can calculate $y_{h,t+1}$, $y_{h,t+2}$, $y_{h,t+2}$ and $y_{h,t+4}$ that make $H_t = 1.03H_0$ for $t = 1$ to 4. Given the technology in the housing sector $y_h = \lambda_h K_h^{\nu} N_h^{1-\nu}$, we can derive the $\lambda_{h,t+1} = 0.5085$, $\lambda_{h,t+2} = 0.50075$, $\lambda_{h,t+3} = 0.5007$, $\lambda_{h,t+4} = 0.50065$ that are consistent with $y_{h,t+1}$, $y_{h,t+2}$, $y_{h,t+3}$ and $y_{h,t+4}$. From the 4th quarter in the experiment, $\lambda_h = 0.5$ and the housing supply gradually come back to the steady state level as houses depreciates.

[24] Notice that the housing down payment is chosen endogenously in the model. Given higher issuance cost (i.e. tighter

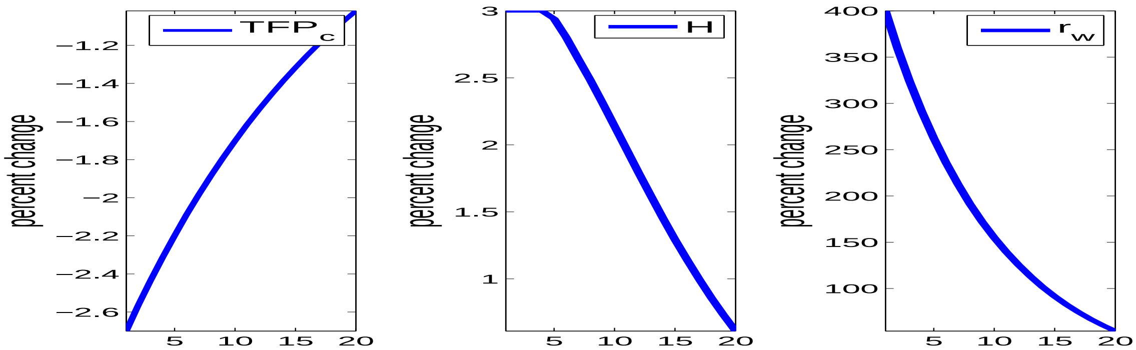

Figure 2: Bechmark Experiment: Paths for All Three Shocks.

Note: All variables are percent change from steady state.

to issuance costs in the mortgage market. The origination fees for mortgage debt was 1.533% from 1990Q1 to 2001Q4 and it declined to 0.569% from 2002Q1 to 2008Q2.[25] With financial innovation such as securitization in the early 2000s, mortgage brokers originated mortgage debt at low cost as interest rate was low and they had weak incentive to assess the borrowers' ability to pay. However, the origination fees increased to 0.691% from 2008Q3 to 2014Q4. The higher origination cost reflects higher approval standards during the financial crisis as screening, monitoring, administration and maintenance costs increase due to increased asymmetric information problems. To mimic the housing financial environment in the Great Recession, banks' cost of issuing mortgage, r_w, unexpectedly increases to 0.0005 in quarter 1 and then evolves according to an AR(1) process with persistence equal to 0.9.[26] The magnitude of the increment is selected such that mortgage debt declines 9% in the baseline experiment to be consistent with the empirical decline in outstanding mortgages during the Great Recession. The persistence is parameter is measured using the quarterly 30 year mortgage origination fees in the data.

5.1 The Baseline Results

In this section, I present the results for the benchmark economy under the three shocks. Figure 3 shows the transitional paths. When the productivity shock hits the economy at $t = 1$, the marginal productivity of capital (MPK) falls which leads to an initial decline in real interest rate. Real wage also falls upon the shocks as the marginal product of labor (MPL) declines for the smaller productivity.[27]

mortgage market), households choose higher down payment. As a result, aggregate mortgage debt declines in equilibrium.

[25]Data source: Freddie Mac.

[26]Parameter r_w evolves according to $r_{w,t+1} = r_w(1 - \rho_r) + \rho_r r_{w,t}$, where $r_{w,1} = 0.0005$.

[27]MPK_c is equal to MPK_h as capital is perfectly mobile. Similarly, MPL_c and MPL_h are equal because labor is perfectly mobile across sectors.

Although capital stock is predetermined when the shocks hits in quarter 1, aggregate output immediately falls. This is a direct consequence of the decline in productivity. Following the initial decline, output decreasing 3.4 percent in quarter 2 for three reasons.[28] First, aggregate capital depreciates slightly as the smaller MPK reduces investment. Second, labor decreases since quarter 2 as real wage continues falling after its initial decline.[29] Third, housing output in quarter 2 becomes much smaller than housing output in quarter 1 as it serves to offset the depreciation to maintain the higher housing supply.

In the period of the shock, real house prices decline about 2.8 percent. We can understand the significant decline in house prices through the supply and demand channel. On the one hand, the productivity shock reduces household asset and labor incomes and hence decreases housing demand. On the other hand, housing over-supply shock generates a 3 percent higher housing stock. To absorb the higher housing supply when housing demand is reduced, house prices need to have a large fall to induce households to purchase more houses.

In the period of the shock, households experience a 1 percent reduction in consumption. In quarter 2, consumption declines 2.2 percent. Consumption remains decreasing until quarter 5 before it starts to recover. The reduction in household income is one factor that reduces consumption. As real interest rate and wage become lower, households receive smaller asset and labor incomes. Moreover, the decline in housing prices contributes to smaller consumption through two channels. First, consumption is reduced through a negative wealth effect: household consume less as they get poorer with smaller housing wealth. Second, as the collateral value of housing declines, households can borrow less to buy the houses. Besides the collateral effects, the tighter mortgage market also reduces household borrowing.

Mortgage debt increases initially, it starts to decline since quarter 2 and remains 9% lower than steady state for four quarters. Household decisions on mortgage debt (and leverage) is determined by four factors together: (a) household demand for housing stock; (b) household income; (c) the real interest rate; (d) the mortgage issuance cost, r_w. It is obvious that the first two factors can affect household demand for mortgage. Moreover, r and r_w determine mortgage demand because they affect the cost and benefit of borrowing through mortgages.[30] On the one hand, since households borrow at the mortgage interest rate and receive interest payments from banks by saving financial assets, the cost of borrowing through mortgages is the net interest payments that households pay out to the banks. On the other hand, the benefit of borrowing is the increase in household value from smoothing consumption. A

[28] The productivity shock takes place in the consumption good sector. Consumption good output falls initially as productivity is smaller. Aggregate output declines significantly because housing output only takes up about 6% of output.

[29] Labor supply increase initially as housing production is labor intensive and housing output increases to realize the 3 percent higher housing stock.

[30] The risk-free interest rate $= r$ while the risk-free mortgage rate $= r + r_w$. So the difference between the mortgage rate and the risk-free saving rate $\geq r_w$.

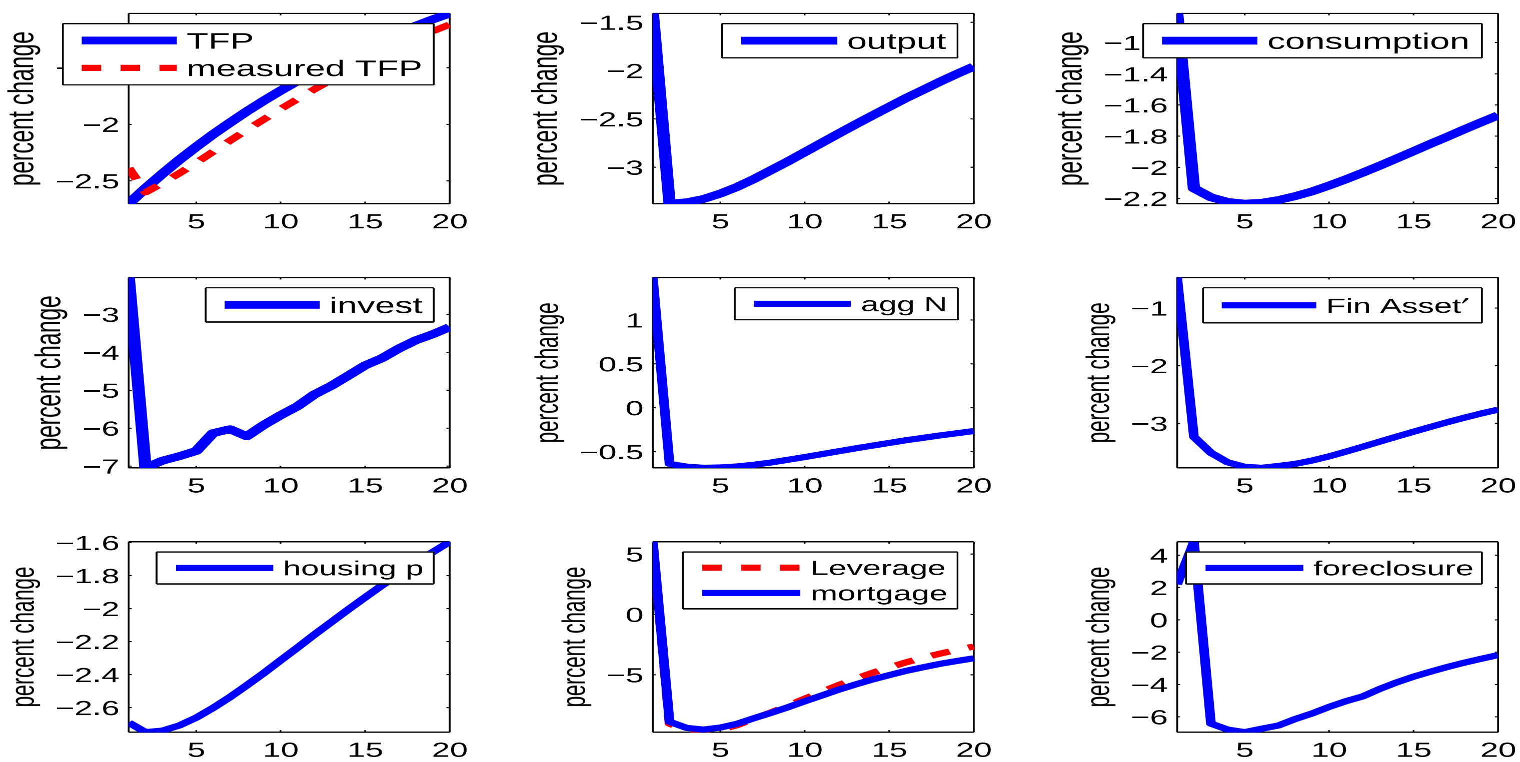

Figure 3: Bechmark Experiment: All Three Shocks.

Note: All variables are percent change from steady state. Endogenous TFP is the red dashed line in the plot 'TFP'. Leverage is the red dashed curve in the plot 'Mortgage'.

larger debt allows household to spend a smaller portion of his saving for the house and thus he can hold larger financial assets after the house is purchased. Since financial assets are not seized by banks when households default, accumulating financial assets enable them to keep their consumption closer to the level before default.

Why does mortgage debt increase in the quarter of the shock? When real house prices decline significantly, household demand for housing asset increases to absorb the excess supply in housing. Thus the demand for mortgage debt (leverage) increases as household purchase more houses. Moreover, real interest rate falls significantly in the first quarter so that the cost of borrowing is reduced even if r_w increases. Thus, households borrow more as mortgage loans are cheaper. Furthermore, lower income provides another incentive for borrowing. Given lower income, they borrow more to smooth consumption.

As real interest rate quickly recovers, the higher mortgage issuance cost starts to play a central role in determining the cost of household borrowing. Given real interest rate, housing h' and housing price p, the change in issuance cost r_w can affect the benefit and cost of borrowing . A higher r_w shifts up the cost of borrowing as households have to make higher net interest payments to the bank for the same (m', h') choice. In the meantime, a higher r_w reduces the benefit of borrowing as households get a smaller loan with the same (m', h') because the loan is now discounted more heavily given a higher r_w. With larger cost and lower benefit, household optimal mortgage becomes much smaller after quarter 3.

Aggregate financial asset declines gradually upon the arrival of the shock. On the one hand, households experiences large declines in housing wealth, labor income and interest income. These factors contribute to lower household wealth and reduce financial assets. On the other hand, households save risk-free financial assets to smooth consumption because the idiosyncratic depreciation shocks might trigger default. Since households take out larger mortgages in the first period, they have incentive to hold more safe assets as their default risk is higher.[31] The two factors make financial assets decline gradually in the economy. Nine quarters after the shocks hit, the effects of these two forces on financial assets are reversed and household financial assets start to recover. On the one hand, households tend to save more financial assets as household income and housing wealth recover. On the other hand, households have incentive to hold less financial asset as their default risks decline.

Foreclosure rate experience a large increase in the first period.[32] The significant drop in real house prices in the first quarter makes a larger fraction of households wake up to find

[31] As house prices decline greatly, household leverage increases as they borrow larger debt in the first two periods. An increase in leverage ι implies a larger default risk as the threshold depreciation rate is equal to $1 - \iota$ and the default probability is equal to $1 - F(1 - \iota)$.

[32] Suppose the banking imbalance caused by unexpected increase in foreclosure does not affect banks to play the basic functions of allocating capital and issuing mortgage.

their housing value fall below the mortgage debt value and thus choose to default. Moreover, foreclosure rate becomes 5.0% higher in the second quarter. One reason default remains high in quarter 2 is because households are poorer and take higher leverage in quarter 1. A second reason is that real house prices are still declining. Foreclosure rate starts to decline gradually since quarter 3 as household take smaller leverage beginning in quarter 2.

In summary, the benchmark experiment with three shocks generates a severe aggregate economic recession, which is accompanied by significant declines in housing prices and mortgage debt. Quantitatively, it explains about 71% of the drop in output, 70% of the reduction in consumption, 15% of the declines in house prices and the entire drop in mortgage debt.

6 The Contributions of Shocks

In this section, I study the contributions of different shocks to the declines in house prices, mortgage debt, output and consumption. When considering the role of one shock, I examine the effects of the other two shocks on the economy and compare them with the results from the baseline experiment. This approach is also applied in Chatterjee and Eyigungor (2015). Actually, when each of these three shocks is the only shock in the economy, the aggregate effects of the shock can hardly have any difference from the marginal contributions discussed below. Thus I mainly present the marginal contributions.

6.1 The Role of Productivity Shock

If the drop in consumption good productivity were eliminated, output would increase initially for higher housing supply and then decrease 0.4% below its steady state in quarter 2 as shown in Figure 13. Output increases initially as more houses are produced and supplied. Consumption falls gradually upon the arrival of the shock and hits the trough (-0.5%) in quarter 2. The decline in consumption is smaller than the baseline experiment for two reasons. First, household experience smaller declines in interest and labor incomes. [33] Second, real house prices only declines by half as much as in the baseline experiment. Thus housing wealth shrink less and the negative wealth effect on consumption is smaller.

If the productivity shock were eliminated, real house prices decline 2.0% upon the arrival of the shock. At the margin, the productivity shock accounts for about 30 % of the decline in house prices in the model. House prices decline less as household income remains relative stable. Specifically, real interest rate falls slightly and real wage actually increases. If there is no change in house prices, housing demand would remain close to its steady state level.

[33] Labor income actually becomes higher. As more houses are supplied, labor increases because the housing sector is labor intensive.

Thus, a smaller decline in house prices is able to have household absorb the excess housing supply. As real house prices experience smaller declines, foreclosure rate increases only 1.6 % in quarter 1.

If there were no productivity shock, mortgage debt would decline about 50% in quarter 2. Without the decline in productivity, real interest rate decreases less in the period of the shock than in the baseline experiment. Therefore the cost of borrowing through mortgage is much higher and hence household reduce borrowing significantly. Moreover, as household income remains stable without the productivity shock, household take smaller leverage.

The productivity shock also contributes to higher foreclosure rate. In its absence, the foreclosure rate would increase 1.6 percent in the period of the shock. From quarter 2, foreclosure decline significantly. Foreclosure rate falls as the increased mortgage issuance cost r_w takes a central role and reduces leverage when the productivity shock is absent. In the baseline experiment, the productivity shock reduces household income and generates higher foreclosure rate because poorer household would borrow more debt and thus have higher default risk. Thus, the lower income resulted from lower productivity indirectly triggers default in the baseline experiment.

If productivity shock in the consumption sector is the only shock to the economy (i.e. holding housing supply and mortgage market unchanged as in steady state), the productivity shock itself is able to generate a 3.0 percent decline in output, 1.9 percent decline in consumption, 20 percent decline in business investment and a 1.0 percent decline in house prices.

6.2 The Role of the Housing Supply Shock

As predicted by many observers, the high housing inventory built before 2006 most likely resulted in a portion of the decline in house prices during the Great Recession. If the housing supply shock were eliminated, house prices would decline 1 percent initially as shown in Figure 14. Thus, the over-supply shock accounts for about two-thirds of the decline in house prices. Moreover, the housing over-supply shock have impact over foreclosure rates. Without the over-supply shock, foreclosure only rises 0.2% initially.

The housing supply shock have smaller contributions to output dynamics than to consumption dynamics. In the absence of the housing supply shock, the trough in output is 0.2 percent smaller while the trough in consumption is smaller by 0.5 percent. The smaller reduction in consumption is because the spillover effect from house prices is smaller without housing over-supply. Given that real house prices decline about 1.0 percent in the first few quarters of the shock, housing wealth shrink less and the negative wealth effect is smaller.

Furthermore, without the housing supply shock, mortgage debt declines 18% immediately when the shocks hit. Mortgage debt has larger decreases than benchmark because housing demand is smaller here. In the baseline experiment, the real house prices declined another 1.8% for households to be willing to take out mortgage debts to buy the excess housing supply. As mortgage debt (and leverage) declines significantly, they hold smaller financial asset as their default risks fall.

6.3 The Role of Mortgage Markets Disruption

In the model, the mortgage markets disruption is accounted for by the higher mortgage issuance cost, r_w. If the tighter mortgage financial condition were eliminated (r_W equals $1.0e{-}4$ as in the steady state), the first major difference from the benchmark is that mortgage debt would increase upon the arrival of the shock as shown in Figure 15. As mentioned earlier, household decision on mortgage debt depends on income, household demand for houses, real interest rate, and mortgage issuance cost r_w. Since the financial condition remains the same as steady state, household have no incentive to reduce borrowing. Instead they have larger demand for mortgage as (i) lower house prices attract household to purchase more houses; (ii) reductions in income generates incentive for borrowing to smooth consumption; (iii) the cost of borrowing declines as real interest rate falls as much as in the baseline experiment. Therefore, households tend to take higher leverage and borrow more debt .

The second major change from the baseline results is that aggregate financial asset increases upon the arrival of the shock. Households hold risk-free financial assets to smooth consumption because the idiosyncratic depreciation shocks might trigger default. Since all households take higher leverage when the shock hits, they insure themselves against higher default risk by holding more safe assets. Therefore, households' demand for financial asset increases upon the shock. As leverage falls when z recovers, households' holding of financial assets follow to decline.

If the mortgage market remains unchanged pre- and post-crisis, foreclosure rate increase persistently as households take higher leverage when they become poorer and buy more houses for the lower prices. One reason default is attractive is that the non-recourse assumption allows households to keep financial assets when defaults happen. A second reason is that defaulters can take out mortgages and purchase houses in the same period they default. The default option is more valuable when mortgages are offered at the same terms pre- and post-crisis.

If the tighter mortgage finance were eliminated, real house prices would have a smaller initial decline, but the difference is small (only 0.1%) from the benchmark experiment. The reason that house prices have similar dynamics with the baseline is that the increase in mortgage debt is mostly used for increasing household financial asset rather than buying houses. Household show strong demand for financial asset because the real interest rate, which is the real rate of return on financial assets, recovers fast after its initial drop. In contrast, the real house prices and housing rental price, which determine the real rate of return on houses, decrease in the first few quarters of the shock before they start to recover. Therefore, household demand for houses here is no stronger than the benchmark experiment.

7 Spillover Effects from the Housing Market

In the previous analysis, we see that when house prices has larger declines in the baseline experiment, the decrease in consumption is also larger. Moreover, when house prices has smaller declines in the experiment without productivity shock (and the experiment without housing supply shock), consumption also have smaller declines. In this section I follow Iacoviello and Neri (2010) to quantify the housing spillover effects on consumption. Focusing on the U.S. economy before the Great Recession (from 1965Q1 to 2006Q3), Iacoviello and Neri (2010) estimate a basic regression that allows for changes in housing wealth to affect consumption, although both variables are endogenous in the model. Following their approach, I estimate the spillover effect by allowing for changes in house prices to affect aggregate consumption. Specifically, I estimate a reduced-form equation that captures the direct and indirect effects that fluctuations in house prices can have on aggregate consumption. Using data from the simulated experiment of my model, the regression of consumption growth on growth in housing prices yields (standard errors are in parenthesis):

$$\Delta lnC_t = \underset{(0.0072)}{-0.0096} + \underset{(0.017)}{0.173}\,\Delta lnP_t \tag{23}$$

The analogous regression using the U.S. data from 2007Q1 to 2015Q4 gives:[34]

$$\Delta lnC_t = \underset{(0.0001)}{0.0000} + \underset{(0.050)}{0.141}\,\Delta lnP_t \tag{24}$$

Comparing the two estimations, we can find that the coefficients of the artificial regression are of similar amount to the coefficients of the data regression. If one is dubious about the sample length of the estimation in (24), the coefficients before the house price growth measured using

[34] Housing data is from FHFA all transaction house price index. Nondurable consumption data is from Table 1.1.5 of the BEA. HPI is seasonally adjusted using ARIMA model. HPI and nondurable consumption are converted to real variables using the GDP deflator. HPI and non-durable consumption are then detrended using Hodrick-Prescott filter with smoothing parameter equal to 1600. The coefficients measured using data from 1975Q1 to 2015Q4 is $\Delta lnC_t = \underset{(0.0002)}{0.000} + \underset{(0.026)}{0.190}\,\Delta lnP_t$.

data from 1975Q1 to 2015Q4 is 0.19 (standard deviation 0.026). Thus the effects of house prices on consumption is again about the same amount to its data counterpart. The positive coefficients for the house price growth indicates a positive correlation between housing prices and consumption. It captures the macroeconomic factors as well as the direct influences of housing wealth on consumption. Specifically, a 1 percent decrease in house prices reduces consumption by 0.17 percent in the model. In the data, a 1 percent decrease in house prices reduces consumption by 0.14 percent over the Great Recession.

8 The Robustness Test

In the baseline experiment, the housing over-supply is achieved through housing over-building that increases the housing stock for 3 percent. Even if the high housing inventory might have contributed to the house prices decline during the Great Recession, the construction of the vacant houses were mainly finished during the run-up to the crisis. This mismatch in the timing of housing construction is because houses take time to build. In this section, we evaluate whether allowing houses to be built during the recession changes the severity and persistence of the aggregate economic recession.

To achieve this evaluation, I compare the aggregate economic results of two experiments which have different settings in housing supply. *Experiment I* is the baseline experiment. *Experiment II* has the productivity shock and the tighter mortgage market in the baseline experiment, but the housing stock is held equal to its steady state level.[35] Comparing the results of the two experiments, I obtain the following findings.

To begin with, the severity and persistence of the aggregate output seem to be independent of the housing supply setting. In both experiments, the decline in output are about equally severe and persistent. Specifically, output falls about 0.26 percent more in *experiment I* than in *experiment II* as shown in Table 4. The small difference in output is because building houses cause reallocation of capital and labor in the two sectors. In terms of persistence, output recovers gradually in both experiments and the half life of the decline in output takes about 25 quarters. The trough is only one quarter later in *experiment I* than experiment II. The dynamics of output is not affected much under different settings of housing supply because the construction sector is relatively small (only 6 percent of output) compare to the aggregate economy.

The persistence of the consumption decline seems to be independent of the housing supply settings. The half life of the consumption decline takes about 35 quarters in both experi-

[35]Housing productivity λ_h is selected to maintain housing stock at its steady state level when the consumption productivity shock and financial tighterning are happening.

Variable	Productivity, Financial and Over-supply Shocks (*Experiment I*)	Productivity and Financial Shocks (*Experiment II*)
output	3.36%, 24 quarters	3.1%, 25 quarters
consumption	2.2%, 35 quarters	1.8%, 35 quarters
house prices	2.8%, 25 quarters	1.1%, 32 quarters

Table 4: The Trough and the Half Life of variables

Note. – The half life of the variable decline refers to the number of quarters it takes for the variable to recover by half as much after it hits the trough. The trough of the variable refers to the magnitude of its decline in the impulse response function.

ments. Moreover, the trough of consumption in both experiments happens around quarter 5. As far as the severity of consumption decline is concerned, there is a 0.4 percent difference in the magnitudes of consumption declines in *experiments I and II.* But if we consider the larger spillover effects from the housing market in *experiment I*, declines in consumption should be 0.3 percent higher (on average) in experiment I than in experiment II. In other words, if we exclude the spillover effects from the housing market, consumption declines in the two experiments would be very close to each other. Therefore, the larger declines in house prices generates larger reductions in consumption, but the construction of houses itself does not seems to affect consumption directly.

As notified by many observers, the productivity and output in the construction sector is declining over the Great Recession. If the contraction in housing output is considered, what would happen to the model economy? To demonstrate this question, I consider a third experiment in which consumption and housing productivity both decline 2.7 percent (endogenous TFP falls about 2.7 percent). After their initial declines, productivity in both the housing and consumption sectors recover according to equation (21). Under this experiment, housing output declines about 8 percent as a direct consequence of the lower productivity. Given smaller housing output, aggregate housing supply declines 2.0 percent. Moreover, output falls 3.3 percent in the period of the shock. The declines in house prices are about 0.3 percent. House prices decline less in this experiment because housing supply is 2 percent smaller than its steady state level. As house prices fall little, consumption only decreases 1.6 percent and the spillover effects from the housing market is negligible.

9 Concluding Remarks

This paper develops a dynamic stochastic general equilibrium (DSGE) model with heterogeneous households and two sectors to explore the impact of tighter mortgage markets, productivity shock and housing over-supply shock on the aggregate economy and housing variables. I calibrate the model to reproduce the housing and non-housing moments in the U.S. data. The resulting economy is characterized by the household behavior of borrowing through mortgage debt and saving using risk-free financial assets and risky housing assets at the same time.

Several experiments have been conducted in order to understand the housing and real economic contractions during the Great Recession. I find that the combination of a productivity shock, a housing over-supply shock and a tighter mortgage market can generate a severe economic recession as well as declines in house prices and mortgage debt. Specifically, the reduction in productivity accounts for most of the declines in output and consumption. Interestingly, productivity shock is important for the fall in house prices and also contributes to the high foreclosure rates. The housing supply shock accounts for two-thirds of house prices decline in the model and also makes the contraction in consumption deeper. House prices has a spillover effect on consumption as household get poorer with smaller housing wealth. Furthermore, the tighter mortgage financial condition explains the large decreases in mortgage debt (and leverage). Households take smaller mortgage debt (and leverage) as the cost of borrowing mortgage to save financial assets have greatly increased.

Future research can first be extended to describe the transmission of the financial crisis. The model abstracts from bank runs that financial intermediaries experience when households default on their mortgage debt. Bank runs might serve as an amplification mechanism for the economy to generate large aggregate contractions. A second extension could consider the time delay in building houses as residential houses usually takes 1-2 years to complete. Because of the delay in construction, the over-supply of houses in the recent recession are mostly completed during the run-up to the crisis. Finally, although my model captures the persistence in house prices declines, the magnitude (2.8 percent) of house prices decline is less than the 19 percent decrease observed in the Great Recession. Thus I speculate some features in the model are responsible for under-predicting the fall in house prices. For one thing, the model may need frictions that break up the link between house prices and productivity. Under the general equilibrium framework with two sectors, the housing prices are determined by the unit cost of housing production, which depends on productivity and interest rate as in equation (10). Specifically, the real house prices are negatively related with the housing productivity and interest rates, but they are positively correlated with the consumption sector productivity. The low interest rate and the reduced productivity observed during the Great Recession are thus hard to reconcile with the plunge of the real house prices if no additional frictions are considered. Moreover, we need to capture why housing demand would remain relatively low over the financial crisis. Housing demand is important in determining house prices given housing supply. However, household housing demand in the model remains relatively high when they experience significant reductions in income. In the model, borrowers buy houses not only because they provide services but also because they can be used as collateral. Household demand houses to borrow, but they are protected from losses by the default option which represents a loss of revenue to lenders. Even though a rental market provides access to housing services without the need to own a house, it is unlikely it will completely eliminate the increase in borrowers' housing demand. Considering the complexity of the current model, I leave these interesting extensions to future research.

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Appendices

A Data

Figure 4 are made using the following data between 1969Q1 and 2014Q4. Output is the Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the BEA. Consumption is Personal Consumption Expenditures less durable goods from Table 1.1.5 of the BEA. Business investment is the sum of durable goods and private nonresidential fixed investment from Table 1.1.5. Total private investment is the sum of business investment and residential investment from Table 1.1.5. TFP is measured using data on total private capital and hours. Hours data are those constructed by Cociuba, Prescott and Ueberfeldt (2012). Total private capital is measured using the perpetual inventory method using the data on total private investment. I set the quarterly depreciation rate 0.0175 and use the 1968Q4 capital as the initial value. All macroeconomic variables are in real terms calculated using the GDP deflator. GDP, consumption, private business investment, measured TFP, outstanding mortgage debt and residential investment are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1969Q1 to 2014Q4. The last 4 data of each variable are now shown to avoid distortion from the HP filter. All real variables are in real terms obtained using GDP deflator, which is measured using Gross Domestic Product (billions of dollars) from Table 1.1.5 divided by Real Gross Domestic Product (billions of chained 2005 dollars) from Table 1.1.6 of the Bureau of Economic Analysis.

I generate Figure 5 and housing moments in the paper using the following data from 1975Q1 to 2015Q4. Housing wealth is the market value of the household owner-occupied real estate including vacant land and mobile homes from the Flow of Funds Accounts of United States. Financial asset is from Table 1169 in the Flow of Funds account. Data on residential investment is from the Table 1.1.5 of the BEA. The house price index is from the FHFA. Quarterly state level house prices are seasonally adjusted purchase only price index from the FHFA. Mortgage debt is obtained from the Board of Governors historical database. Housing foreclosure rate is reported in the National Delinquency Survey from 2002Q1 to 2013Q4 by the Mortgage Banker Association. Loan-to-value is measured using real mortgage debt divided by real housing wealth. All real variables are in real terms obtained using GDP deflator.

B Solution Methods

B.1 Consumption-Leisure-Savings Problem

Following Jeske, Krueger and Mitman (2013), household's problem in this model can be transformed into the following consumption-leisure-savings problem. Let $\hat{c}$ denote the total consumption over consumption good and housing services, i.e., $\hat{c} = c + p_s s$.

$$v(x, \epsilon) = \max_{\hat{c}, g', n} u(\hat{c}, 1 - n; p_s) + \beta \sum_{\epsilon'} \pi(\epsilon'|\epsilon)\omega(g', \epsilon') \tag{25}$$

subject to

$$\hat{c} + g' = w\epsilon n + x \tag{26}$$

$$0 \leq n < 1, \quad g' \geq 0 \tag{27}$$

where $\omega(g', \epsilon')$ is given by

$$\omega(g', \epsilon') = \max_{h', m', a' \geq 0} \int_{\underline{\delta}}^{1} v(x', \epsilon') dF(\delta') \tag{28}$$

s.t.

$$x' = (1 + r')a' + \max\{0, p'(1 - \delta')h' - m'\} \tag{29}$$

$$g' = a' + (p - p_s)h' - m' p_m(m', h') \tag{30}$$

In order to solve the consumption-leisure-savings problem, I need to solve $\omega(g', \epsilon')$ first. The optimization problem represented by equations (26)-(28) that solves $\omega(g', \epsilon')$ is called the household portfolio problem. The portfolio problem involves selecting houses, financial assets and mortgage debt to maximize household expected lifetime discounted value.

B.2 The Steady State

I approximate the continuous *log AR(1)* process for labor productivity with a 5 state Markov chain using the procedure in Tauchen and Hussey (1991). The five labor productivity realizations are $\{0.4244, 0.6659, 1.0000, 1.5018, 2.3563\}$ and the transition matrix is

$$\Pi = \begin{pmatrix} 0.7629 & 0.2249 & 0.0121 & 0.0001 & 0.0000 \\ 0.2074 & 0.5566 & 0.2207 & 0.0152 & 0.0001 \\ 0.0113 & 0.2221 & 0.5333 & 0.2221 & 0.0113 \\ 0.0001 & 0.0152 & 0.2207 & 0.5566 & 0.2074 \\ 0.0000 & 0.0001 & 0.0121 & 0.2249 & 0.7629 \end{pmatrix}$$

1. Building grid points for net worth x and net saving g. I use 23 log-spaced points over $[0, 45]$ for g, 33 log-spaced points over $[0, 45]$ for x. The result of the model remains about the same with a more intensive grid.

2. Set the guesses of the real interest rate r and the real rental price p_s.

3. Given r and p_s, do the following to find the value function and optimal decision rules:

(i) Set a guess for value function $v(x, \epsilon)$

(ii) Solve the household portfolio problem and find $\omega(g, \epsilon)$ for each (g, ϵ) on the grid.

(iii) Solve the consumption-leisure-savings problem represented by equation (24)-(26) to update value function and find the optimal net saving g^* for each (x, ϵ) on the grid.

(iv) Given $g^*(x, \epsilon)$, find the optimal asset decision rules $h^*(x, \epsilon)$, $m^*(x, \epsilon)$ and $a^*(x, \epsilon)$ using the decision rules solved in (ii).

4. Compute the steady state distribution of individuals over net worth x, idiosyncratic productivity ϵ and default choice d, $\mu(x, \epsilon, d)$.

(i) I use 750 evenly spaced grid points over $[0, 45]$ for x and the same grid points chosen in step 1 for ϵ.

(ii) Approximate the housing, asset, mortgage decision rules solved in Step (3) using splines. For households starting with (x, ϵ, d), use the decision rules to determine the cutoff depreciation rate $\delta^* = 1 - \frac{m'}{p'h'}$. Then $1 - F(\delta^*)$ fraction out of $\mu(x, \epsilon, d)$ defaults and $F(\delta^*)$ fraction repay.

(iii) For people starting with (x, ϵ, d), they have the same δ^* and decision rules, but they might end up with different net worth next period since housing depreciation δ is idiosyncratic. I simulate δ draws for households at (x, ϵ, d) using the Pareto distribution and calculate their future net worth x' according to their δ' draws.

(iv) Suppose future net worth is such that $x_j \leq x' \leq x_{j+1}$ for a individual at (x, ϵ, d), where x_j and x_{j+1} are two adjacent net worth grid points. Then update μ assuming that individuals at (x, ϵ, d) move to (x_j, ϵ', d) with probability $\varphi\pi_\epsilon(\epsilon'|\epsilon)$ and to (x_{j+1}, ϵ', d) with probability $(1 - \varphi)\pi_\epsilon(\epsilon'|\epsilon)$, where $\varphi = \frac{x_{j+1} - x}{x_{j+1} - x_j}$. Update μ until it converges. The resulting μ is the fixed point of household distribution $\bar{\mu}$

5. With $\bar{\mu}$, calculate aggregates in the economy and solve the production problems. Then check market clearing conditions. If rental market and consumption good market do not clear, update guesses of r and p_s and come back to step 3.

B.3 Transitional Dynamics

The transitional dynamics is solved through the following three steps:

(1) Start with steady state value function and solve the value function backwardly from $t = T - 1$ to $t = 1$

(2) Start with steady state distribution, update the household distribution forwardly from $t = 1$ to $t = T$

(3) Check the market clearing conditions in each period to update interest rate and rental price.

The length of time $T = 160$ in the negative productivity shock and $T = 270$ in the Great Recession Experiment and the Foreclosure Experiment. $T = 120$ in the depreciation shock experiment. Since households in this model are forward looking, they look at current as well as future prices whey they make decisions. When the period t prices are updated, they affect aggregates in the markets in period $t - 1$ and t. Therefore, I use a small parameter 0.001 to update prices. The precision for value function $= 1.0e - 4$. The precision for household distribution $= 1.0e - 8$. The precision for market clearing conditions in the steady state and the transitional dynamics $= 1.0e - 3$. All other precision $= 1.0e - 6$.

B.4 Solving the Production Problems

First order conditions for the consumption good sector:

$$[K_c] \quad \alpha z K_c^{\alpha-1} N_c^{1-\alpha} = r + \delta_k \tag{31}$$

$$[N_c] \quad (1-\alpha) z K_c^{\alpha} N_c^{-\alpha} = w \tag{32}$$

First order conditions for the housing good sector:

$$[N_h] \quad (1-\nu) p \lambda_h K_h^{\nu} N_h^{-\nu} = w \tag{33}$$

$$[K_h] \quad \nu p \lambda_h K_h^{\nu-1} N_h^{1-\nu} = r + \delta_k \tag{34}$$

Equation (31) and (32) imply

$$w = \left(\frac{1-\alpha}{\alpha}\right) \left[\alpha z \left(\frac{1}{r+\delta_k}\right)^{\alpha}\right]^{\frac{1}{1-\alpha}} \tag{35}$$

$$\frac{K_c}{N_c} = \left(\frac{\alpha}{1-\alpha}\right) \left(\frac{w}{r+\delta_k}\right) \tag{36}$$

Equation (33) and (34) imply

$$p = \frac{1}{\lambda_h \nu} \left(\frac{\nu}{1-\nu}\right)^{1-\nu} (r+\delta_k)^{\nu} w^{1-\nu} \tag{37}$$

$$\frac{K_h}{N_h} = \left(\frac{\nu}{1-\nu}\right)\left(\frac{w}{r+\delta_k}\right) \tag{38}$$

Recall the labor market and the capital market clearing conditions

$$K_c + K_h = \bar{K}, \quad N_c + N_h = \bar{N} \tag{39}$$

Together can solve

$$K_c = \frac{\left(\frac{1-\nu}{\nu}\right)\bar{K} - \left(\frac{w}{r+\delta_k}\right)\bar{N}}{\frac{1-\nu}{\nu} - \frac{1-\alpha}{\alpha}} \tag{40}$$

$$K_h = \frac{\left(\frac{w}{r+\delta_k}\right)\bar{N} - \left(\frac{1-\alpha}{\alpha}\right)\bar{K}}{\frac{1-\nu}{\nu} - \frac{1-\alpha}{\alpha}} \tag{41}$$

Using equation (34) and (36) can obtain

$$p = \frac{1}{\nu\lambda_h}\left(\frac{\nu}{1-\nu}\right)^{1-\nu}\left(\frac{1-\alpha}{\alpha}\right)^{1-\nu}(\alpha z)^{\frac{1-\nu}{1-\alpha}}\left(\frac{1}{r+\delta_k}\right)^{\frac{\alpha-\nu}{1-\alpha}} \tag{42}$$

$$w = (\alpha z)^{\frac{1}{1-\alpha}}\left(\frac{1-\alpha}{\alpha}\right)\left(\frac{1}{r+\delta_k}\right)^{\frac{\alpha}{1-\alpha}} \tag{43}$$

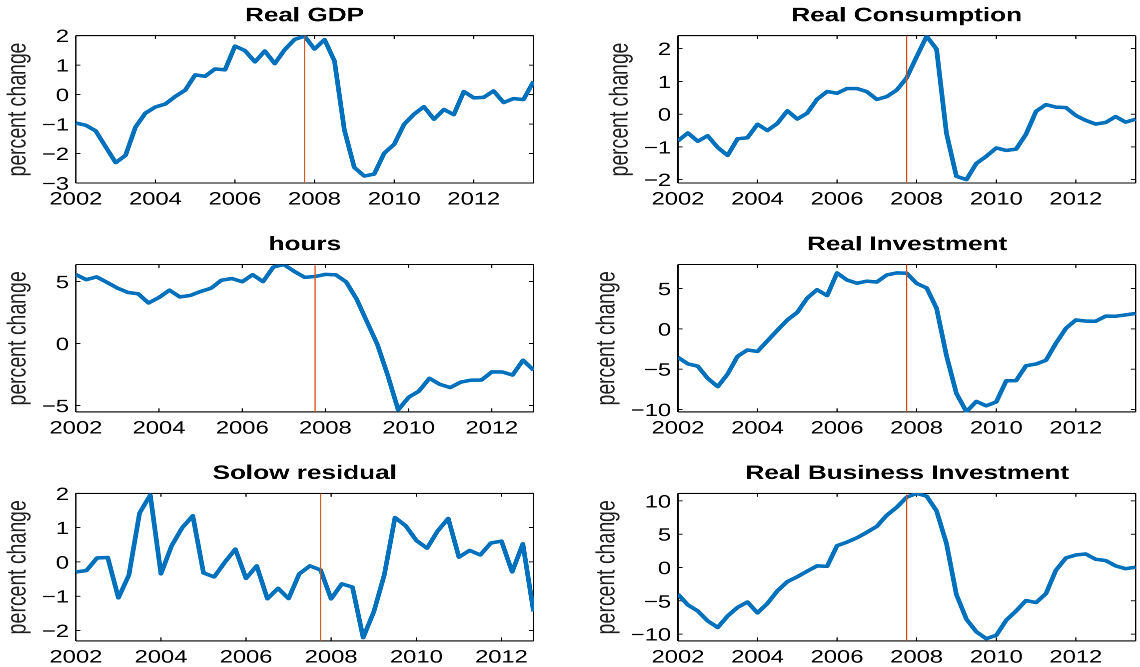

Figure 4: U.S. Business Cycle Facts.

NOTE. – Vertical line is the starting date (2007Q4) of the Great Recession. Real GDP, consumption, private and business fixed investment series are from NIPA Table 1.1.5 of the BEA. Consumption is nondurable goods and services; private investment is the sum of business fixed investment, residential investment and consumer durables. Total hours is civilian and military hours worked by noninstitutional population aged 16 to 64 from Cociuba et. al (2012). Solow Residual is a direct calculation using the capital and labor shares to which our model is calibrated, with private capital taken from Fixed Assets Tables of the BEA. All series are in logs, detrended using the Hodrick-Prescott filter with smoothing parameter 1600, and plotted as percentage deviations from mean values; the series are constructed using data from 1969Q1 - 2014Q4.

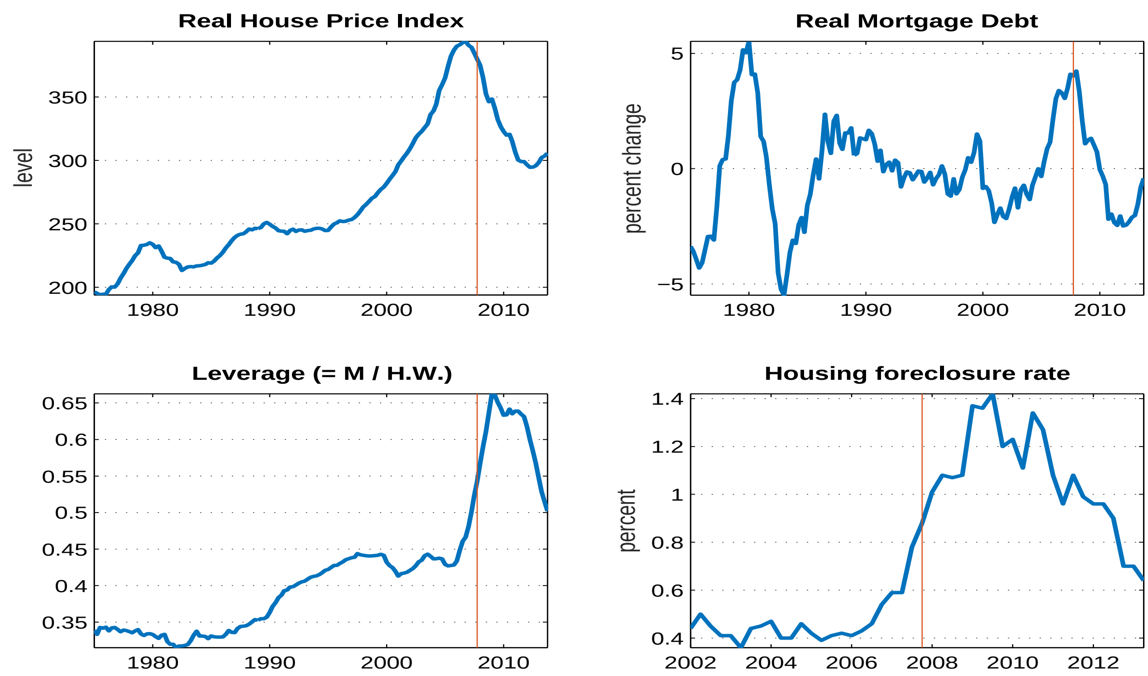

Figure 5: U.S. Housing Market Facts.

NOTE. – Vertical line is the starting date (2007Q4) of the Great Recession. HPI is sales only index from FHFA. Mortgage Debt is the outstanding single-family mortgage debt from the Board of Governors Historical Database. Housing wealth is from Flow of Funds Table Z1. Foreclosure rate is quarterly starts from the Mortgage Banker Association and is reported in percentage point. Series are converted into real values using the GDP deflator. Leverage (Loan-to-Value) is a numerical ratio. House prices and mortgage debt are detrended using Hodrick-Prescott filter with a smoothing parameter of 1600 from 1975Q1 to 2015Q4. The figure shows the percent deviation from their trends.

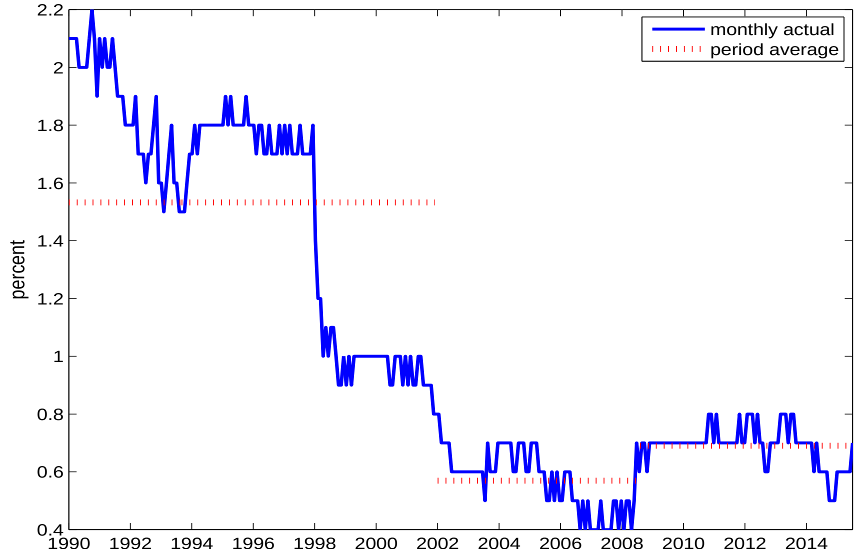

Figure 6: Mortgage Origination Fees and Discount Points for 30-Year Fixed Rate Mortgage in the United States. Origination fees and discount points are the total charged by the lender at settlement. One point equals one percent of the loan amount. *Data source: Freddie Mac*

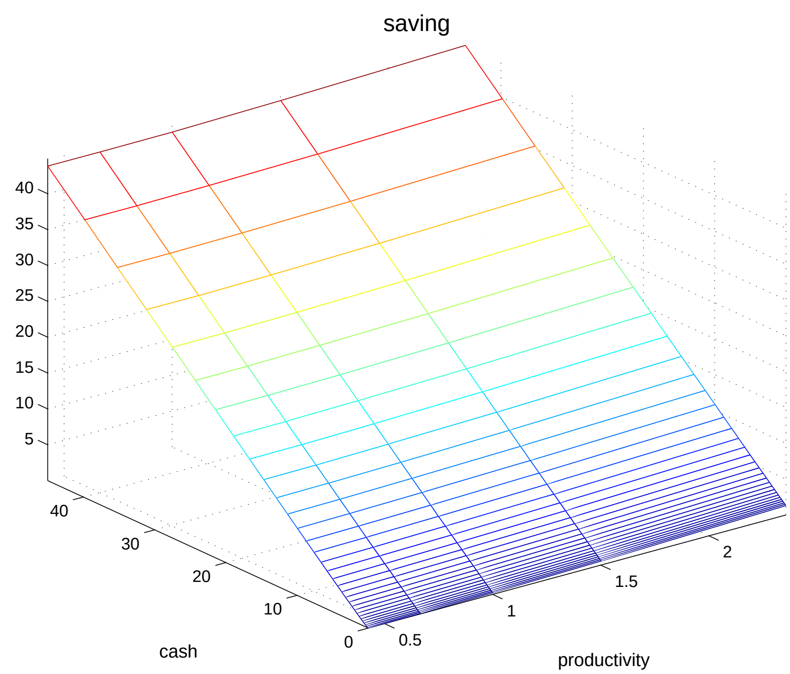

Figure 7: Net Saving Policy

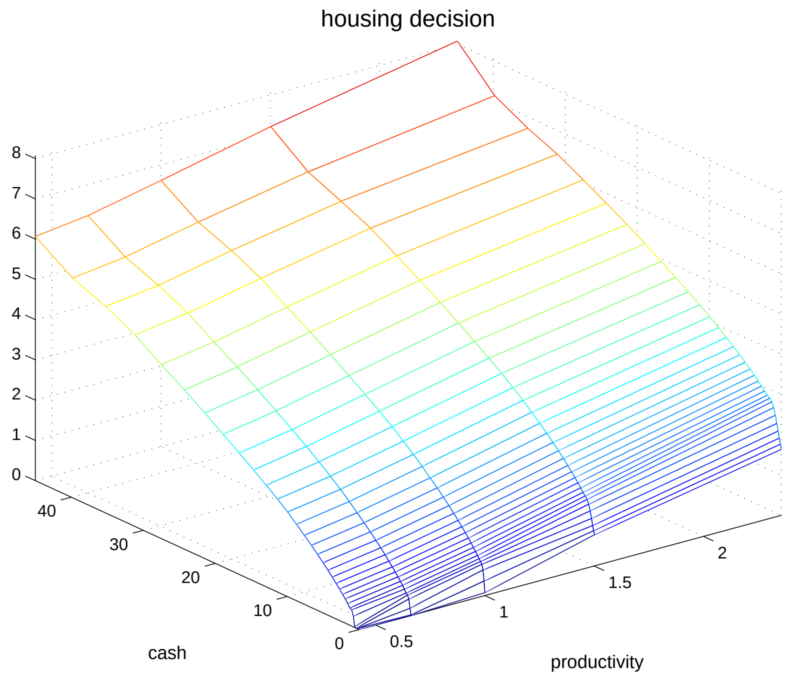

Figure 8: Housing Asset Policy Function

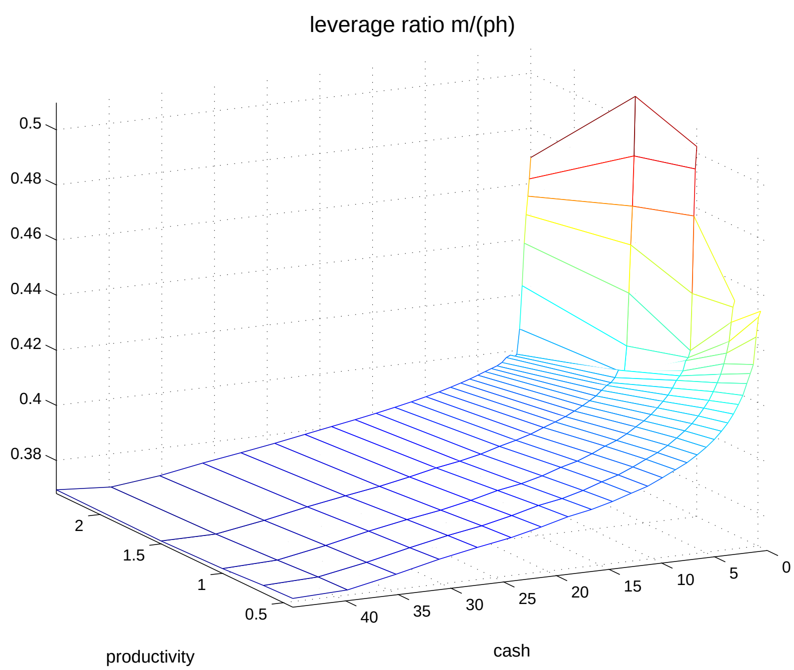

Figure 9: Household Leverage Policy, l=m/(ph)

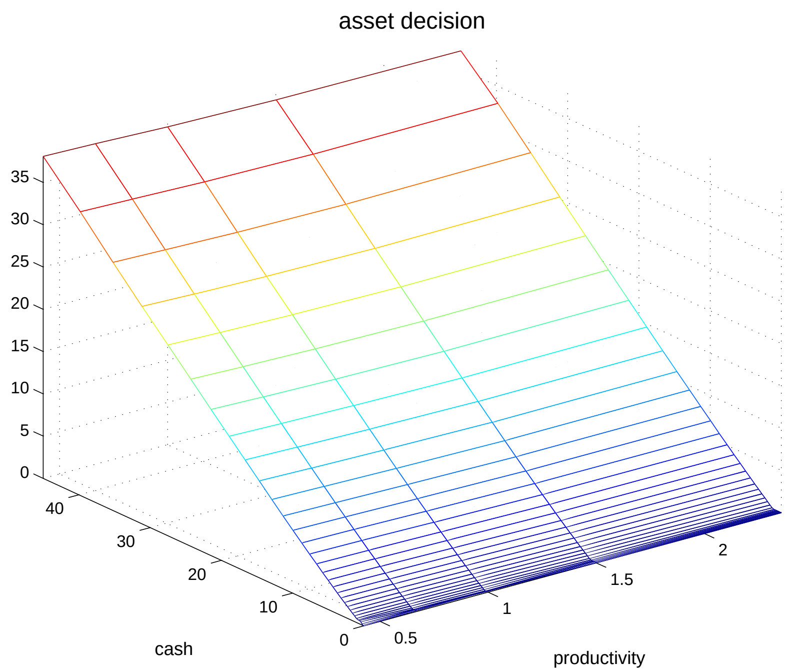

Figure 10: Financial Asset Policy

Figure 11: Labor Supply Policy

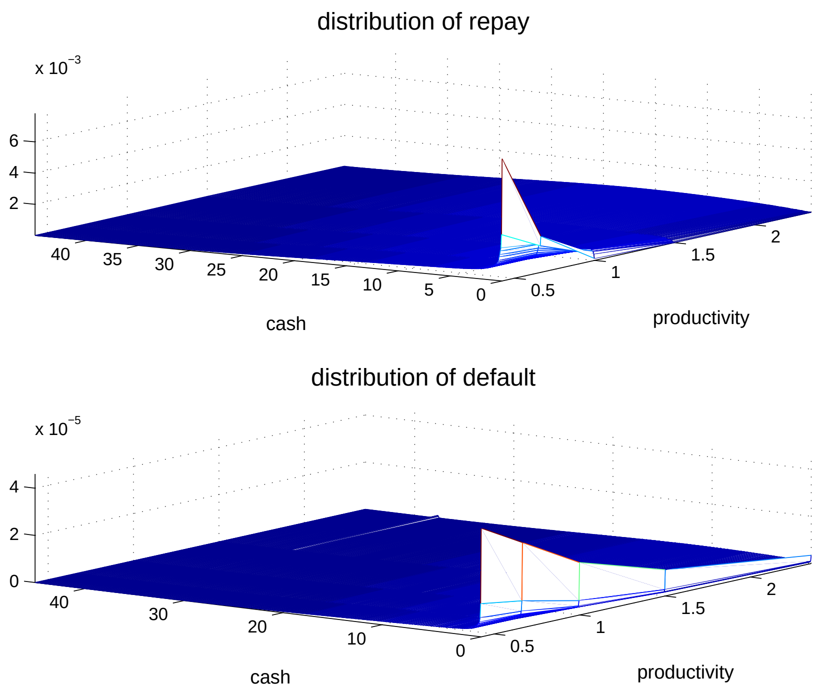

Figure 12: Households distribution

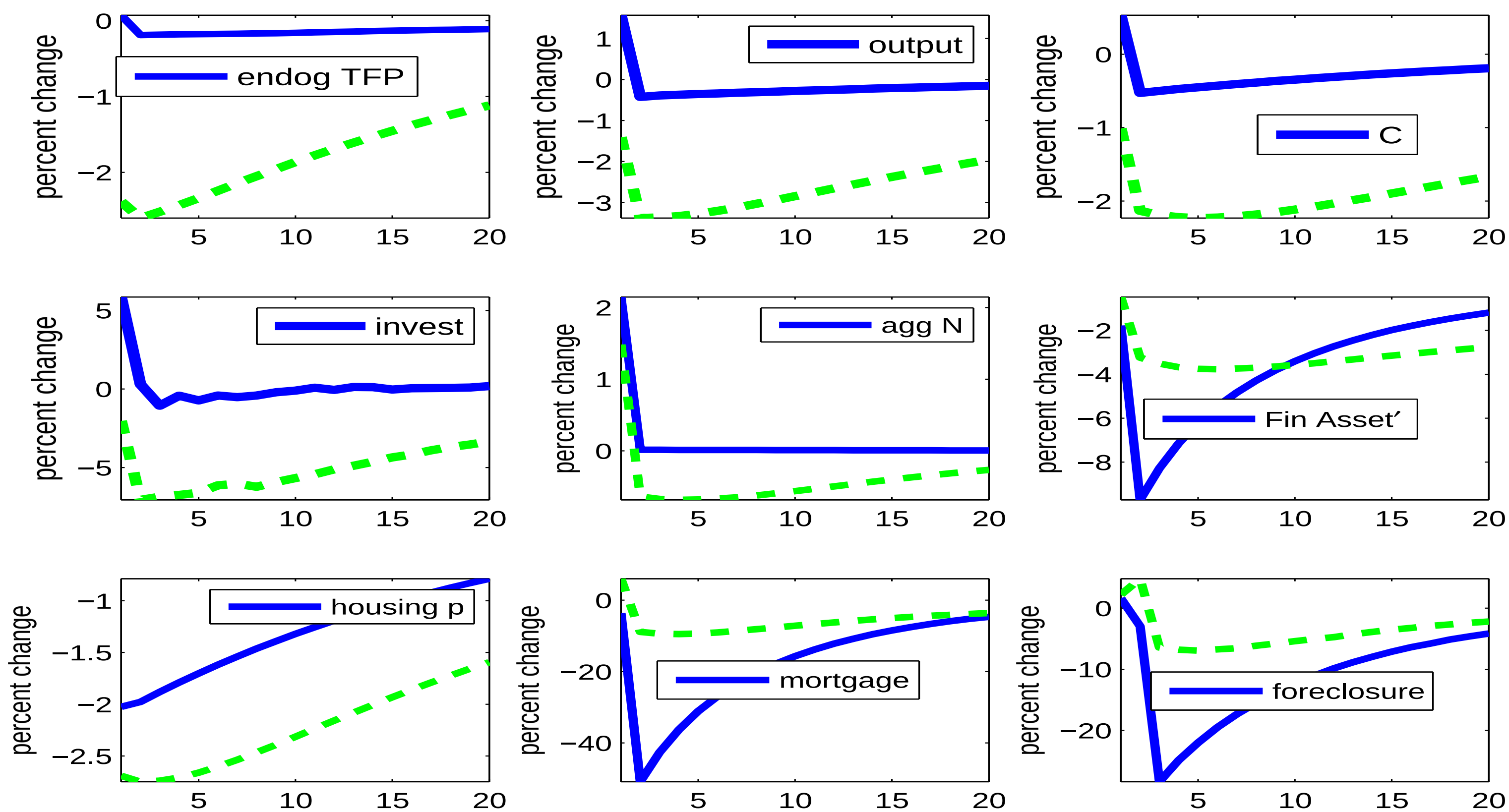

Figure 13: No Productivity Shock

Note: The solid blue line is for the no productivity shock case. The dashed green line is for the benchmark experiment. All variables are percent change from steady state.

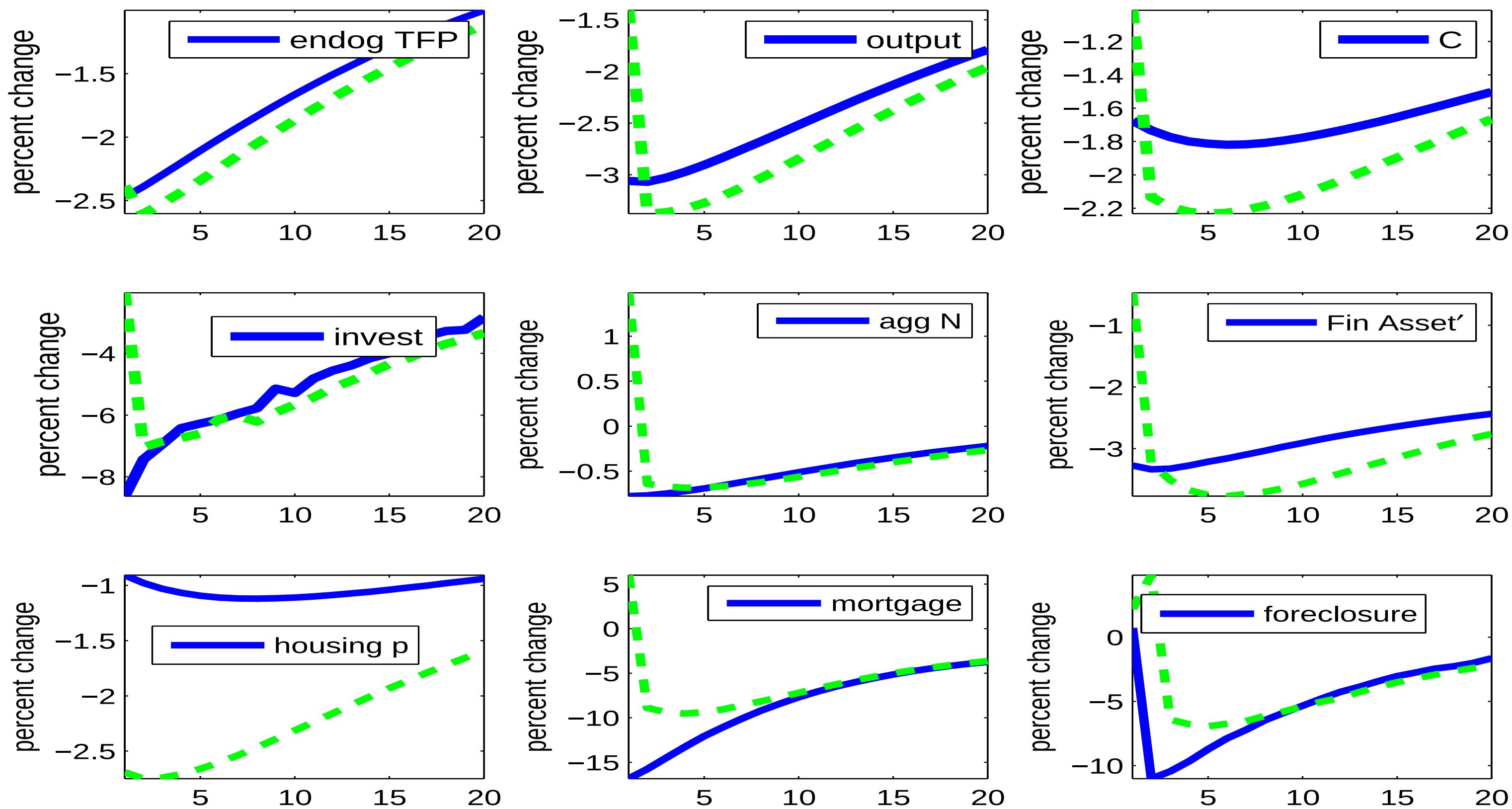

Figure 14: No Housing Over-Supply.

Note: The solid blue line is for the no housing over-supply case. The dashed green line is for the baseline experiment. All variables are percent change from steady state.

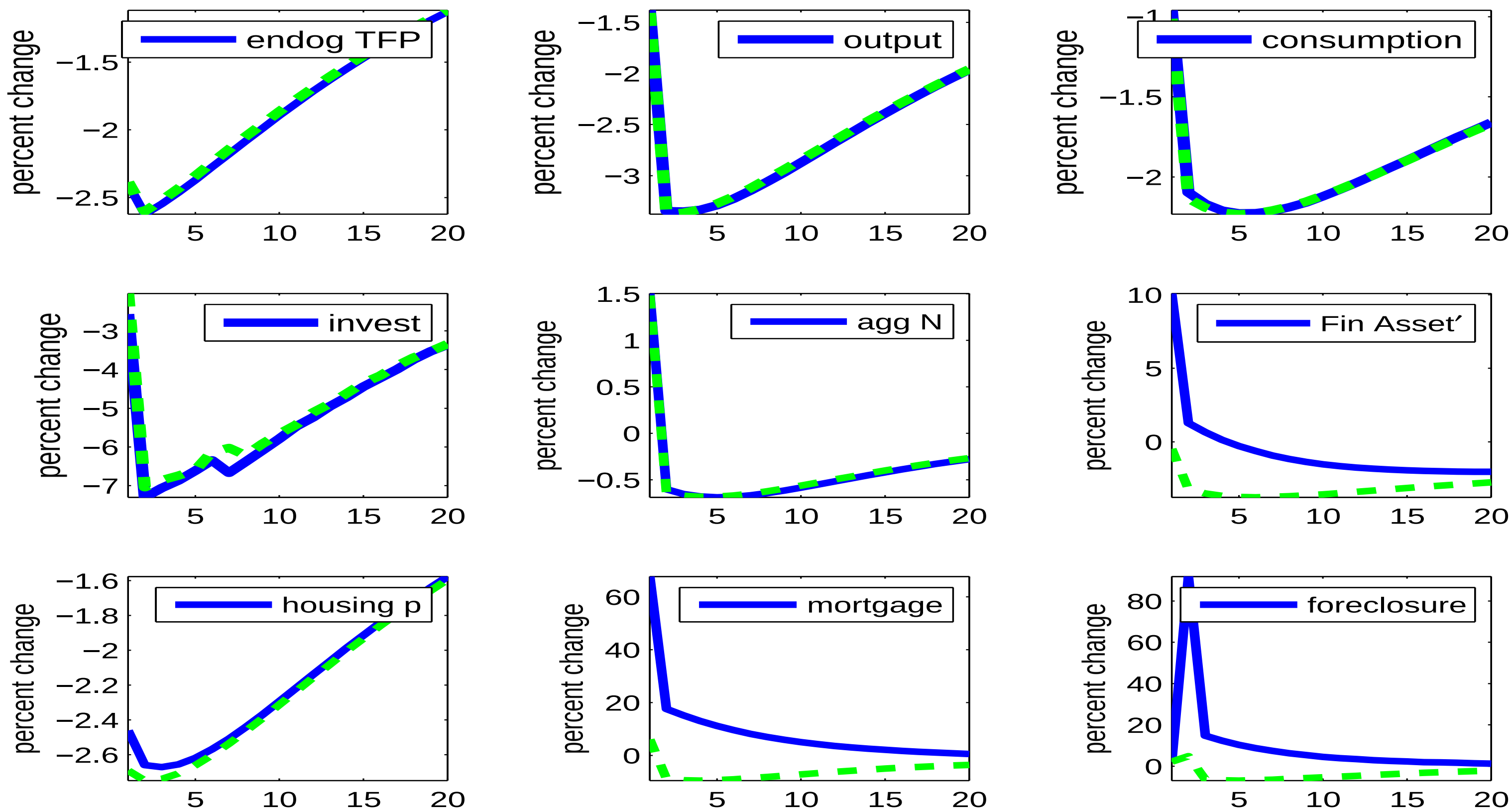

Figure 15: No Tighter Mortgage Market

Note: The solid blue line is the no tighter mortgage market case. The dashed green line is for the baseline experiment. All variables are percent change from steady state.